Solving Linear Inequalities

Inequalities are useful for comparing things **that are not equal**. In this handout, we will focus on solving linear inequalities.

The chart below lists the symbols used in inequalities, their verbal equivalent, and an example where x represents an unknown value.

Symbol	Verbal Description	Example	
<	less than	x < 2	
>	greater than	x > -1	
≤	less than or equal to	x ≤ 1	
≥	greater than or equal to	x ≥ -3	
≠	does not equal to	x ≠ 3	

Filled dots are used when x is "less than or *equal* to" (≤) AND "greater than or *equal* to" (≥) the boundary value. Filled dots include the value underneath the dot. **Open circles** are used when x is "less than" (<) AND "greater than" (>) the boundary value. Open circles do NOT include the value underneath.

<u>Solving Inequalities</u>

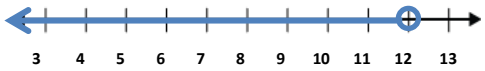
When we are asked to ***solve*** an inequality, our goal is to find all of the ***solutions*** of the inequality that make the statement true. In other words, we are finding all of the values that can be substituted into the unknown variable that make it true. <u>To do this, we need to isolate for the unknown variable.</u>

Rules for Solving Inequalities

1. We can **add** or **subtract** a number from both sides of an inequality.
2. We can **multiply** or **divide** both sides of an inequality by a **positive number**.

3. We can **multiply** or **divide** both sides of an inequality by a **negative number**, if we **change the direction of the inequality sign**.

Example 1: Solve the inequality $x - 3 < 9$.

$x - 3 < 9$	
$x - 3 \mathbf{+ 3} < 9 \mathbf{+ 3}$	Add **+3** to both sides of the inequality.
$x < 12$	

Thus, all values that are less than 12 can be substituted into x for $x - 3 < 9$ to be true.

CHECK! Determine whether x = 11 holds true.

1. Sub x = 11 into the linear inequality.
2. Simplify both sides.
3. Since 8 is less than 9 the statement holds true.

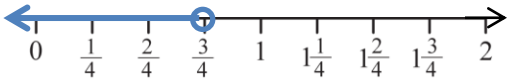
$$x - 3 < 9$$
$$\mathbf{11} - 3 < 9$$
$$8 < 9$$

Example 2: Solve the inequality $x + 6 \geq 2$.

$x + 6 \geq 2$	
$x + 6 \mathbf{- 6} \geq 2 \mathbf{- 6}$	Subtract **6** from both sides of the inequality.
$x \geq -4$	

Thus, all values that are greater than and equal to -4 can be substituted into x for $x + 6 \geq 2$ to be true.

Example 3: Solve the inequality $4x + 2 < 5$.

$4x + 2 < 5$	
$4x + 2 \mathbf{- 2} < 5 \mathbf{- 2}$	Subtract **2** from both sides of the inequality.
$4x < 3$	
$\frac{4x}{\mathbf{4}} < \frac{3}{\mathbf{4}}$	Divide both sides by **4.**
$x < \frac{3}{4}$	

Recall Rule 3: When we **multiply** or **divide** both sides of an inequality by a **negative number**, we need to **change the direction of the inequality sign**.

Let's look at a number line to understand why…

1. On the number line, we see the inequality, **4 < 8** holds true. $4 < 8$
2. Let's divide the inequality on both sides by -1. $\frac{4}{-1} < \frac{8}{-1}$
3. Then, the inequality fails since -4 is not less than -8. $-4 < -8$
4. To make the statement hold true, we need to switch the direction of the inequality sign. $-4 > -8$

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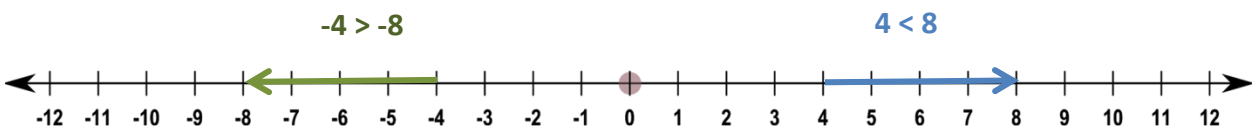


Example 4: Solve the inequality -2x ≤ 4.

-2x ≤ 4	
$\frac{-2x}{-2} \leq \frac{4}{-2}$	Divide both sides of the inequality by **-2.**
x **≥** -2	Switch the direction of the inequality sign.
	

Sometimes we may be asked to solve two inequalities at once. These types of inequalities are solved in the same way with the added step of applying each operation to both sides.

Example 5: Solve the inequality $3 < \frac{(6-3x)}{2} < 6$.

$-3 < \frac{(6-3x)}{2} < 6$	
$-3(\mathbf{2}) < \frac{(6-3x)}{2}(\mathbf{2}) < 6(\mathbf{2})$	Multiple both sides of the inequality by **2**.
-6 < (6 – 3x) < 12	
-6 **– 6** < (6 – 3x) – 6 < 12 **– 6**	Subtract **6** from both sides of the inequality.
-12 < -3x < 6	

$\frac{-12}{-3} < \frac{-3x}{-3} < \frac{-6}{-3}$	Divide both sides of the inequality by **-3**.
$4 > x > -2$	Switch the direction of the inequality sign.
	

Exercises:

1. Solve the following inequalities and plot the solutions on a number line:

   a) $3x + 1 > 0$

   b) $-5x \leq 10$

   c) $8 < -2x + 4 \leq 3$

2. Solve the following inequalities.

a) $3x - 12 > 15$

b) $-1 \leq -4x + 15 \leq 7$

c) $\frac{1}{2}x - 5 \leq 6x$

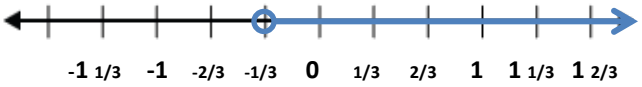
d) $3 > 10x - 7 > 23$

e) $-\frac{1}{4}x + 8 < 6$

f) $3x - \frac{4}{5} \geq -x$

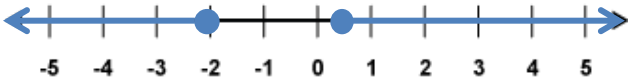
Solutions:

1. a) $x > -1/3$



b) $x \geq -2$

c) $-2 > x \geq \frac{1}{2}$



2.

a) $x > 9$

b) $4 \geq x \geq 2$

c) $x < -8$

d) $1 > x > 3$

e) $x > 8$

f) $x \geq \frac{1}{5}$