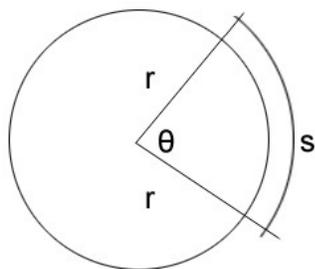


Radian Measure

To assign a radian measure to an angle θ , consider θ to be a central angle of a circle of radius 1, as shown in Fig.3.1. The radian measure of θ is then defined to be the length of the arc s of the sector. Because the circumference of a circle is $2\pi r$ the circumference of a unit circle (of radius 1) is 2π . This implies that the radian measure of an angle measuring 360° is 2π . In other words, **$360^\circ = 2\pi$ radians**.



The arc length s of the sector is the radian measure of θ .

Arc length is: $s = r \theta$

Fig.1.1

The graphs below, Fig 3.2 and Fig 3.3 show the degrees of the **unit circle** in all 4 quadrants, from 0° to 360° and the relationship between radians and degrees respectively.

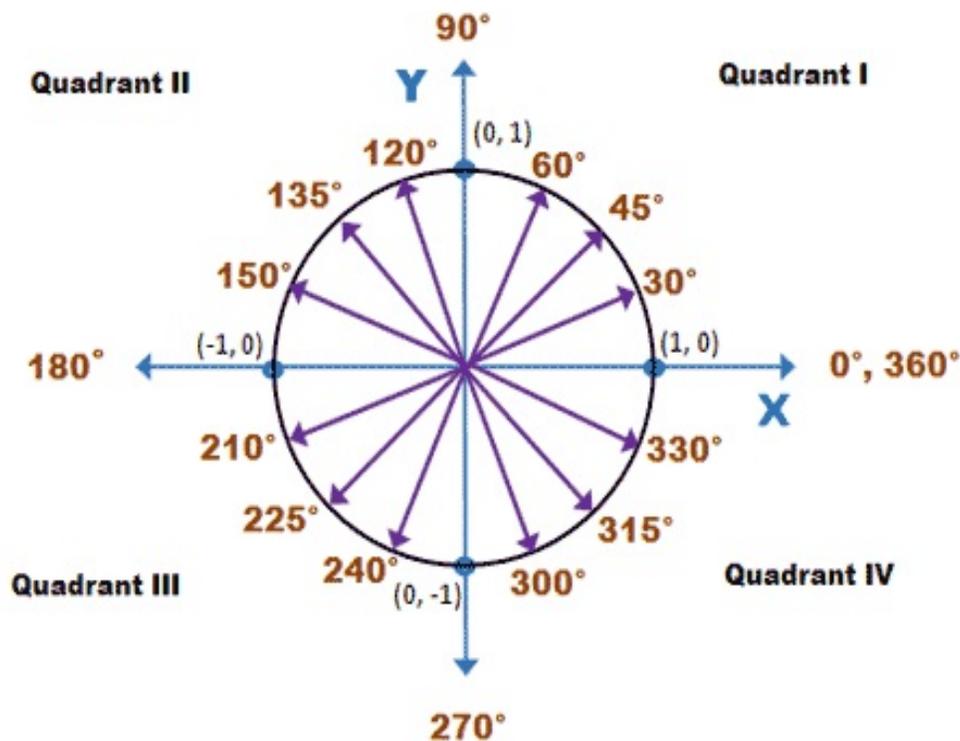


Fig.1.2

Radian Measure

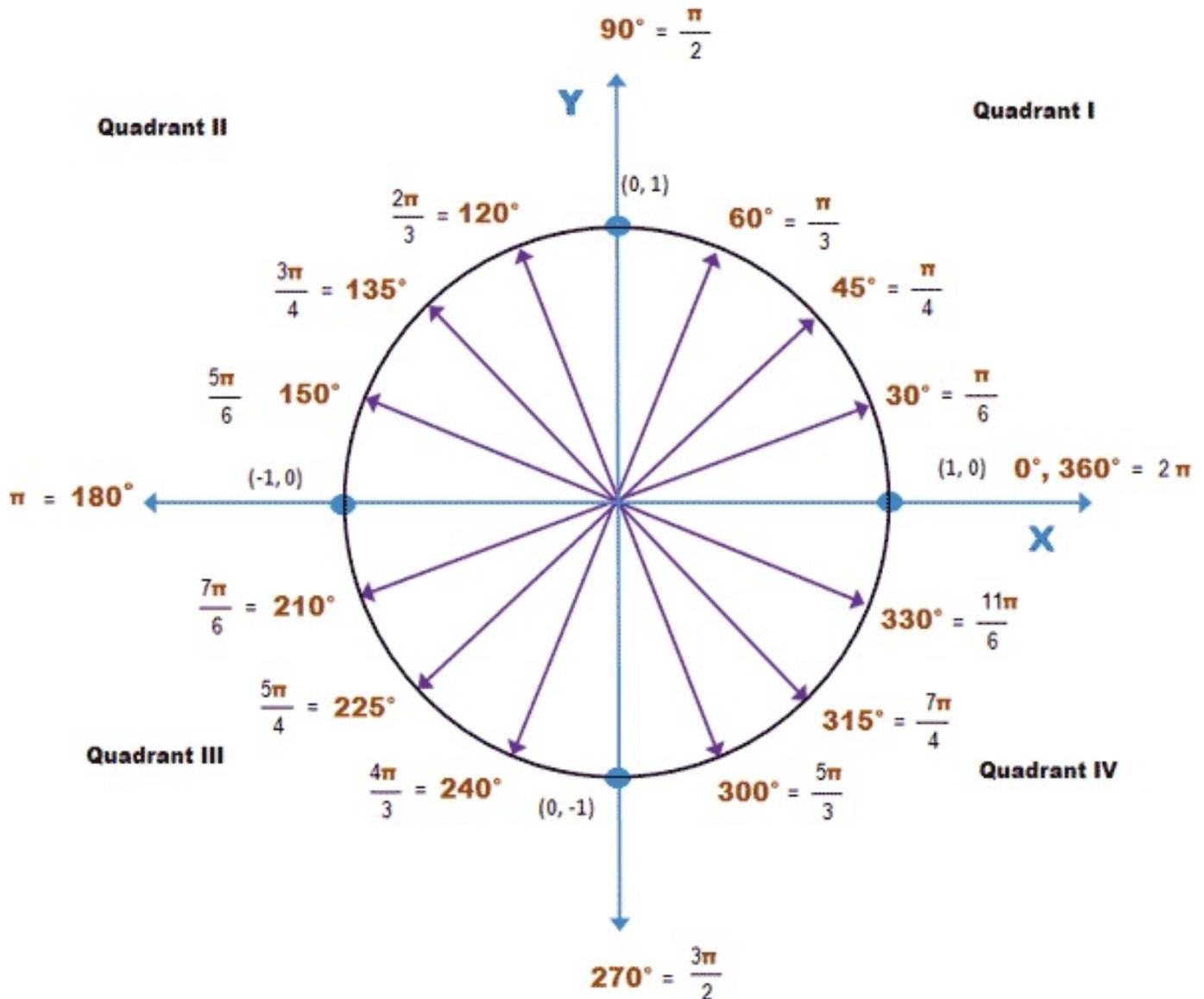


Fig 1.3

Radians is the standard unit of angle measure. The formula for calculating Radians is:

$$\text{Radians} = (\text{Degrees}) \times \left(\frac{\pi}{180^\circ} \right)$$

Radian Measure

Example 1:

Convert the following degrees into radians.

Degrees	Formula	Radians	Simplified
0°	$(0^\circ) * (\pi/180^\circ) =$	0	0
30°	$(30^\circ) * (\pi/180^\circ) =$	30 π /180 radians	$\pi/6$
45°	$(45^\circ) * (\pi/180^\circ) =$	45 π /180 radians	$\pi/4$
60°	$(60^\circ) * (\pi/180^\circ) =$	60 π /180 radians	$\pi/3$
90°	$(90^\circ) * (\pi/180^\circ) =$	90 π /180 radians	$\pi/2$
120°	$(120^\circ) * (\pi/180^\circ) =$	120 π /180 radians	2 π /3
135°	$(135^\circ) * (\pi/180^\circ) =$	135 π /180 radians	3 π /4
150°	$(150^\circ) * (\pi/180^\circ) =$	150 π /180 radians	5 π /6
180°	$(180^\circ) * (\pi/180^\circ) =$	180 π /180 radians	π
210°	$(210^\circ) * (\pi/180^\circ) =$	210 π /180 radians	7 π /6
225°	$(225^\circ) * (\pi/180^\circ) =$	225 π /180 radians	5 π /4
240°	$(240^\circ) * (\pi/180^\circ) =$	240 π /180 radians	4 π /3
270°	$(270^\circ) * (\pi/180^\circ) =$	270 π /180 radians	3 π /2
300°	$(300^\circ) * (\pi/180^\circ) =$	300 π /180 radians	5 π /3
315°	$(315^\circ) * (\pi/180^\circ) =$	315 π /180 radians	7 π /4
330°	$(330^\circ) * (\pi/180^\circ) =$	330 π /180 radians	11 π /6
360°	$(360^\circ) * (\pi/180^\circ) =$	360 π /180 radians	2 π /1 = 2 π

Practice Exercise:

Find the radian measure of the angles with the given degree measures.

- a) 108° d) -30° g) -142.5°
b) 18° e) 1080° h) 40°
c) -150° f) -192° i) -100°

Answer:

- a) $3\pi/5$ d) $-\pi/6$ g) $-28.5\pi/36$
b) $\pi/10$ e) 6π h) $2\pi/9$

Radian Measure

c) $-5\pi/6$ f) $-16\pi/15$ i) -1.8π

Practice Exercise 2:

Find the degree measure of the angles with the given radian measures.

a) $11\pi/6$ d) -1.3 g) $-\pi$
b) $-5\pi/4$ e) $\pi/10$ h) 40π
c) 4 f) $-7\pi/15$ i) -2π

Answer:

a) 330° d) -74.5° g) -180°
b) -225° e) 18° h) 7200°
c) 229.2° f) -84° i) -360°

Example 2:

Find the coterminal angles for the following radian measures of angles:

a) $2\pi/3$
b) $23\pi/4$

Solution:

a) $2\pi/3 + (2\pi) = 8\pi/3$
 $2\pi/3 + 2(2\pi) = 4\pi + 2\pi/3 = 14\pi/3$
 $2\pi/3 + 3(2\pi) = 6\pi + 2\pi/3 = 20\pi/3$
b) $23\pi/4 = 5\pi + 3\pi/4$

The coterminal angles are therefore, $2\pi n + 3\pi/4$, for all positive integer n.

Example 3:

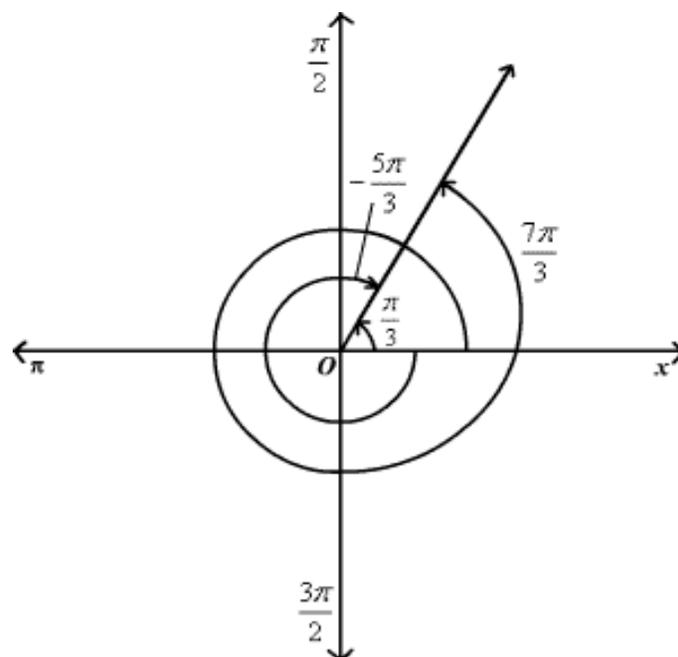
Find a positive and a negative angle coterminal angle with a $\pi/3$ angle.

Solution:

A positive coterminal angle with $\pi/3$ is $\pi/3 + 2\pi = 7\pi/3$.
(Generally a positive coterminal angle to $\pi/3 = \pi/3 + 2\pi n$, for $n > 0$)

A positive coterminal angle with $\pi/3$ is $\pi/3 - 2\pi = -5\pi/3$.
(Generally a negative coterminal angle to $\pi/3 = \pi/3 - 2\pi n$, for $n > 0$)

See figure to the right.



Radian Measure

Practice Exercise 3:

The measure of an angle in standard position is given. Find two positive angles and two negative angles that are coterminal with the given angle. (State your answers as a comma-separated list.)

- 1) 350°
- 2) 450°
- 3) -200°
- 4) -5°
- 5) $7\pi/4$
- 6) $-\pi/4$
- 7) $2\pi/3$
- 8) $-5\pi/2$

Answer:

1. $-10^\circ, -370^\circ, 710^\circ, 1070^\circ$
2. $-270^\circ, -360^\circ, 90^\circ, 810^\circ$
3. $-560^\circ, -920^\circ, 160^\circ, 520^\circ$
4. $-725^\circ, -365^\circ, 355^\circ, 715^\circ$
5. $-\pi/4, -9\pi/4, 15\pi/4, 23\pi/4$
6. $-17\pi/4, -9\pi/4, 15\pi/4, 23\pi/4$
7. $-10\pi/6, -4\pi/6, 8\pi/3, 14\pi/3$
8. $-\pi/2, -9\pi/2, 3\pi/2, 7\pi/2$