Introduction to Bonds

Bonds are a debt instrument, where the bond holder pays the issuer an initial sum of money known as the **purchase price**. In turn, the issuer pays the holder **coupon payments** (annuity), and a final sum (**face value**) upon maturity of the bond. Bonds can be thought of as printed contracts. The terms of the contract include:

- **Face value** (a.k.a. **par value** or **denomination**) – the amount owed to the holder of the bond when it matures.
- **Bond rate** (a.k.a. **coupon rate** or **nominal rate**) – the rate of interest paid based on the face value of the bond (bond rates are usually paid semi-annually). The rate is used to determine the periodic interest payments paid out during the term of the bond.
- **Maturity date** (a.k.a. **due date** or **redemption date**) – the date on which the principal of the bond is to be repaid.
- **Principal** (a.k.a. **maturity value** or **redemption value**) – the amount paid by the issuer to the bondholder when the bond is surrendered.

Most bonds are **redeemable at par** (i.e. redeemed at their face value).

Some bonds are **callable** and can be redeemed prior to the maturity date. These types of bonds are **redeemable at premium** (i.e. value **greater** than the face value of the bond). The redemption value is stated as a percentage of face value. For example, a $1000 bond redeemable at 105 is redeemed at 105% of $1000 = $1050.

The value of a bond on a particular date includes two main components (promises):

1. The dated value of the principal (or face value)
2. The dated value of the sum of the period interest payments (an annuity)

Bonds can be freely bought and sold. When an investor buys bonds, there are two main calculations to consider:

1. For a given rate of return, what should be the purchase price of the bond?
2. If purchased at a given price, what is the rate of return to the investor?

The purchase price of a bond can be calculated using the following formula:

\[
\text{Purchase price} = \text{PV of the redemption price} + \text{PV of the sum of interest payments}
\]

\[
\text{Purchase price} = FV(1 + p)^{-n} + PMT \left(1 - (1 + p)^{-n}\right) \div p
\]

where \( p = (1 + i)^c - 1 \)
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FV = principal of the bond
PMT = periodic interest payment (a.k.a. coupon)
\[ \text{PMT} = \text{Face value} \times \text{bond rate} \]
n = the # of outstanding interest payments (or compounding periods)
i = the yield rate per payment interval

Note: Two types of interest rates are used to determine the purchase price of a bond:
- The bond rate determines the size of the PMT.
- The **yield rate** determines the present value of the two promises of a bond.

When bonds are sold, there are three possible situations:

1. If the bond rate (b) = market rate (i), the bond sells at **par**.
2. If the bond rate (b) < market rate (i), the bond sells at **a discount**. In this case, the purchase price of the bond is **less than** the principal.
3. If the bond rate (b) > market rate (i), the bond sells at **a premium**. In this case, the purchase price of the bond is **greater than** the principal.

The **premium** or **discount** on the purchase of a bond can be calculated using the following formula:

\[
\text{Premium or discount} = (b \times \text{face value} - i \times \text{redemption price}) \left[1 - \left(1 + \frac{i}{n}\right)^{-n}\right] \div i
\]

Sample Problem 1:

A $2500 bond pays interest at 8% semi-annually and is redeemable at par at the end of 5 years. Determine the purchase price to yield a holder, if the bond pays 10% compounded annually.

**Method 1: Using Formula**

\[
\begin{align*}
\text{FV} &= 2500 \\
\text{PMT} &= 2500 \times 0.08/2 = 100 \\
i &= 0.1 \div 2 = 0.05 \\
n &= 5 \times 2 = 10
\end{align*}
\]

\[
P \text{ (purchase price)} = \text{FV}(1+i)^{-n} + \text{PMT} \left(1-(1+i)^{-n}/i\right)
\]
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\[= 2500(1+ 0.05)^{-10} + 100(1−(1+ 0.05)^{-10}/0.05)\]

\[= $1534.78 + $772.17\]

\[= $2306.95\]

**Method 2: Using Financial Calculator**

Step 1: Calculate the present value of redemption price.

P/Y=2
C/Y=2
FV=2500
N=5 x 2=10
I/Y=10

PV of redemption price = −1534.78

Step 2: Calculate the present value of the interest payments.

PMT= 2500 x 0.08/2 = 100

P/Y=2
C/Y=2
FV=0
N=5 x 2=10
I/Y=10

PV of interest payments = −772.17

Purchase price = $1534.78 + $772.17 = $2306.95

*Note: This bond was purchased at a discount, since the purchase price is less than maturity value.*

**Sample Problem 2**

A $5000 bond maturing at 105 on September 1, 2031, has semi-annual coupons at 7%. Determine the purchase price on March 1, 2010 to guarantee a yield of \( j_2 = 6.8\% \).

**Method 1: Using Formula**

Step 1: Calculate the coupon (PMT) amount

\[\text{PMT} = \text{Face value} \times \text{coupon rate}\]
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\[ \text{PMT} = 5000 \times 0.07 \div 2 = 175 \]

Step 2: Calculate face value of the bond. Remember that the bond is redeemable at a premium of 105%.

\[ \text{FV} = 5000 \times 1.05 = 5250 \]

Step 3: Calculate the purchase price of the bond.

\[ n = 43 \]
\[ i = 0.068 \div 2 = 0.034 \]

\[ P \text{ (purchase price)} = \text{FV}(1+i)^{-n} + \text{PMT}(1-(1+i)^{-n}/i) \]
\[ = 5250(1+0.034)^{-43} + 175(1-(1+0.034)^{-43}/0.034) \]
\[ = 5171.50 \]

**Method 2: Using Financial Calculator**

Step 1: Calculate the present value of the face value of the bond.

\[ \text{P/Y} = \text{C/Y} = 2 \]
\[ \text{FV} = 5000 \times 1.05 = 5250 \]
\[ \text{N} = 43 \]
\[ \text{I/Y} = 6.8 \]
\[ \text{PV} = ? \]

\[ \text{PV} = 1246.74 \]

Step 2: Calculate the present value of the interest payments.

\[ \text{PMT} = 5000 \times 0.07/2 = 175 \]
\[ \text{P/Y} = \text{C/Y} = 2 \]
\[ \text{N} = 43 \]
\[ \text{I/Y} = 6.8 \]
\[ \text{FV} = 0 \]
\[ \text{PV} = ? \]

\[ \text{PV} = 3924.76 \]

Purchase price = $1246.74 + $3924.76 = $5171.50

*Note: The bond is purchased at a premium, since the purchase price is greater than the maturity value.*
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Practice Problems

Practice Questions:

1. A 10-year $2000 bond pays semi-annual coupons at $j_2=6\%$ and is purchased to yield $j_2=6\%$. What is the purchase price if the bond is redeemable at
   a) Par
   b) 101.5


3. A $1000 bond bearing semi-annual coupons at is redeemable at par. What is the minimum number of whole years that the bond should run so that a person paying $1100 for it would earn at least $j_2= 8\%$?

4. A corporation issues $600 000 worth of 12-year bonds with semi-annual coupons at $j_2=10\%$. The bonds are priced to yield $j_2=9\%$. Determine the issue price per $100 unit.

Answers:

1. a) $2000
   b) $2016.61

2. $4523.70

3. 7 years

4. $107.25