SIMPLE INTEREST

\[ I = Prt \]
- \( I \) is the amount of interest earned
- \( P \) is the principal sum of money earning the interest
- \( r \) is the simple annual (or nominal) interest rate (usually expressed as a percentage)
- \( t \) is the interest period in years

\[ S = P + I \]
\[ S = P (1 + rt) \]
- \( S \) is the future value (or maturity value). It is equal to the principal plus the interest earned.

COMPOUND INTEREST

\[ FV = PV (1 + i)^n \]

\[ i = \frac{j}{m} \]
- \( j = \) nominal annual rate of interest
- \( m = \) number of compounding periods
- \( i = \) periodic rate of interest

\[ PV = FV (1 + i)^{-n} \quad \text{OR} \quad PV = \frac{FV}{(1 + i)^n} \]

ANNUITIES

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### Ordinary Simple Annuity

\[ FV_n = PMT \left( \frac{(1+i)^n - 1}{i} \right) \]

**Note:** \( \frac{(1+i)^n - 1}{i} \) is called the **compounding** or **accumulation factor for annuities** (or the accumulated value of one dollar per period).

\[ PV_n = PMT \left( \frac{1 - (1+i)^{-n}}{i} \right) \]

### Ordinary General Annuity

\[ FV_g = PMT \left( \frac{(1+p)^n - 1}{p} \right) \quad PV_g = PMT \left( \frac{1 - (1+p)^{-n}}{p} \right) \]

***First, you must calculate \( p \) (equivalent rate of interest per payment period) using \( p = (1+i)^{c} - 1 \) where \( i \) is the periodic rate of interest and \( c \) is the number of interest conversion periods per payment interval.

\[ c = \frac{\text{# of interest conversion periods per year}}{\text{# of payment periods per year}} \]

\[ c = \frac{C}{P} \]

### Constant Growth Annuity

- **size of \( n \)th payment = \( PMT \cdot (1+k)^{n-1} \)**
- **\( k \) = constant rate of growth**
- **\( PMT \) = amount of payment**
- **\( n \) = number of payments**

\[ \text{sum of periodic constant growth payments} = PMT \left( \frac{(1+k)^n - 1}{k} \right) \]

\[ FV = PMT \left( \frac{(1+i)^n - (1+k)^n}{i-k} \right) \]
\[
\left[\frac{(1+i)^n - (1+k)^n}{i-k}\right] \text{ is the compounding factor for constant – growth annuities.}
\]

PV = PMT \left[\frac{1-(1+k)^n(1+i)^{-n}}{i-k}\right]

\[
\frac{1-(1+k)^n(1+i)^{-n}}{i-k} \text{ is the discount factor for constant – growth annuities.}
\]

PV = n (PMT) (1 + i)^t \text{ [This formula is used when the constant growth rate and the periodic interest rate are the same.]}

**SIMPLE annuity DUE**

\[FV_{n\text{(due)}} = PMT \left[\frac{(1+i)^n - 1}{i}\right] (1 + i)\]

\[PV_{n\text{(due)}} = PMT \left[\frac{1-(1+i)^n}{i}\right] (1 + i)\]

**GENERAL annuity DUE**

\[FV_g = PMT \left[\frac{(1+p)^n - 1}{p}\right] (1 + i)\]

\[PV_g = PMT \left[\frac{1-(1+p)^n}{p}\right] (1 + i)\]

***Note that you must first calculate \( p \) (equivalent rate of interest per payment period) using \( p = (1+i)^c - 1 \) where \( i \) is the periodic rate of interest and \( c \) is the number of interest conversion periods per payment interval.

**ORDINARY DEFERRED ANNUITIES or DEFERRED ANNUITIES DUE:**

Use the same formulas as ordinary annuities (simple or general) OR annuities due (simple or general). Adjust for the period of deferment – period between “now” and the starting point of the term of the annuity.

**ORDINARY SIMPLE PERPETUITY**

\[PV = \frac{PMT}{i}\]

**ORDINARY GENERAL PERPETUITY**

\[PV = \frac{PMT}{p} \text{ where } p = (1+i)^c - 1\]
SIMPLE PERPETUITY DUE

\[ PV \text{ (due)} = \text{PMT} + \frac{PMT}{i} \]

GENERAL PERPETUITY DUE

\[ PV \text{ (due)} = \text{PMT} + \frac{PMT}{p} \]
where \( p = (1+i)^c - 1 \)

AMORTIZATION involving SIMPLE ANNUITIES:

Amortization refers to the method of repaying both the principal and the interest by a series of equal payments made at equal intervals of time.

If the payment interval and the interest conversion period are equal in length, the problem involves working with a simple annuity. Most often the payments are made at the end of a payment interval meaning that we are working with an ordinary simple annuity.

The following formulas apply:

\[ PV_n = \text{PMT} \left[ \frac{1-(1+i)^{-m}}{i} \right] \]
\[ FV_n = \text{PMT} \left[ \frac{(1+i)^m-1}{i} \right] \]

Finding the outstanding principal balance using the retrospective method:

Outstanding balance = FV of the original debt – FV of the payments made

Use \( FV = PV \ (1 + i)^n \) to calculate the FV of the original debt.

Use \( FV_n = \text{PMT} \left[ \frac{(1+i)^m-1}{i} \right] \) to calculate the FV of the payments made