Effective Interest Rates

The nominal rate is the interest rate as stated, usually compounded more than once per year. The effective rate (or effective annual rate) is a rate that, compounded annually, gives the same interest as the nominal rate. If two interest rates have the same effective rate, we say they are equivalent.

To find the effective rate \( f \) or a nominal rate \( j \) compounded \( m \) times per year, we can use the formula

\[
f = \left(1 + \frac{j}{m}\right)^m - 1
\]

Using a BAII Plus calculator, we can determine the effective rate in the following way:

2nd 2 (ICONV)
Enter the nominal rate, then press ENTER↓↓, then enter the number of compounding periods per year, and press ENTER↑↑CPT and the effective rate will be displayed.

Example 1
Suppose we want to find the effective rate of an investment at 9% compounded quarterly.

Formula:

\[
f = \left(1 + \frac{0.09}{4}\right)^4 - 1 = (1.0225)^4 - 1 = 0.09308 = 9.31\%
\]

BAII Plus: 2nd 2 9 ENTER ↓↓ 4 ENTER ↑↑ CPT Display: EFF= 9.308331879

So, the effective rate of 9% compounded quarterly is approximately 9.31%.

Example 2
What interest rate, compounded quarterly, has an effective rate of 15%?

Formula:

\[
0.15 = \left(1 + \frac{j}{12}\right)^{12} - 1
\]

Rearranging to find \( j \), we get

\[
j = 12 \times \left(\frac{1}{12}\right) \left(1 + 0.15\right)^{1/12} - 1
\]

\[
j = 0.1406
\]
Effective Interest Rates

**BAII Plus**: `2nd 2] 15 ENTER [ 12 ENTER [ ] CPT`  
Display:  
NOM=14.0579003

So 14.06% compounded quarterly has an effective rate of 15%.

**Sample Exercises**

1. Find the effective annual rate of  
   a. 8.5% compounded quarterly  
   b. 4% compounded monthly  
   c. 5.8% compounded annually  
   d. 7.25% compounded semi-annually  
   e. 12.5% compounded monthly

2. You can make a one-year investment at 7.8% compounded monthly, or 8% compounded semi-annually. Which option should you choose?

3. What nominal rate, compounded quarterly, is equivalent to an effective annual rate of 10%?

4. What nominal rate has an effective rate of 8%, compounded  
   a. Semi-annually?  
   b. Quarterly?  
   c. Monthly?

**Solutions**

1. a. 8.77%  
   b. 4.07%  
   c. 5.8%  
   d. 7.38%  
   e. 13.24%

2. The effective rate of 7.8% compounded monthly is 8.08%.  
The effective rate of 8% compounded semi-annually is 8.16%.  
You should choose to invest at 8% compounded semi-annually.

3. We know $0.10 = \left(1 + \frac{j}{4}\right)^4 - 1$, so rearranging we get  
   \[ j = 4 \times \left(\left(1 + 0.10\right)^\frac{1}{4} - 1\right) \]  
   \[ j = 0.0965 \]

4. a. 7.85%  
   b. 7.77%  
   c. 7.72%