ACCUPLACER Elementary Algebra Assessment Preparation Guide

Please note that the guide is for reference only and that it does not represent an exact match with the assessment content. The Assessment Centre at George Brown College is not responsible for students' assessment results.

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Introduction

The Accuplacer Elementary Algebra Assessment Preparation Guide is a reference tool for George Brown College students preparing to take the Accuplacer Elementary Algebra assessment. The study guide focuses on foundation-level math skills. **The study guide does not cover all topics on the assessment.**

The Accuplacer Elementary Algebra Assessment Preparation Guide is organized around a select number of topics in algebra. It is recommended that users follow the guide in a step-by-step order as one topic builds on the previous one. Each section contains theory, examples with full solutions and practice question(s). Answers to practice questions can be found in Appendix B.

Reading comprehension and understanding of terminology are an important part of learning mathematics. Appendix A has a glossary of mathematical terms used throughout the Study Guide.

1. Evaluating algebraic expressions

To evaluate an algebraic expression:

- Substitute the numerical values of variables into the algebraic expression putting ***brackets*** around the numerical values.
- Evaluate the expression using the correct order of operations.

Order of Operations

As a *general* rule of thumb, the acronym BEDMAS can be followed for the correct order of operations.

Brackets

Exponents — Do in order from left to right

Division
Multiplication — Do in order from left to right

Addition
Subtraction — Do in order from left to right

Important notes:

- If there are **multiple exponents**, evaluate the powers from left to right as they appear in the question.
- If there are **multiple brackets**, it does not matter which ones you do first, second, etc.
- If there are **multiple operations *within* brackets**, do the operations according to BEDMAS.
- **Division and multiplication** is done from left to right. This means that multiplication should be done before division, if it appears to the left of division.
- **Addition and subtraction** is done from left to right. Subtraction should be done first, if it is to the left of addition.
- For **rational expressions**, the numerator and denominator are evaluated separately according to BEDMAS. Then, determine the quotient.

Example

Evaluate $-5(-3x^3 + 18y)$ when $x = 3$ and $y = -5$.

$-5[-3(3)^3 + 18(-5)]$ Step 1: Substitute (3) for x and (–5) for y in the algebraic expression.

Step 2: Evaluate the expression using the correct order of operations.

$= -5[-3(3)^3 + 18(-5)]$	a) Evaluate the exponent first.
$= -5[-3(27) + 18(-5)]$	b) Do the multiplication inside the brackets.
$= -5[-81 + (-90)]$	c) Do the addition inside the brackets.
$= -5(-171)$	d) Do the multiplication.
$= 855$	Final answer.

Practice questions:

1. **Evaluate $(17k^4 + 19y)^2 - 13$ when $k = -1$ and $y = 5$**
   - **a)** 6071
   - **b)** 12531
   - **c)** 716
   - **d)** 143

2. **Evaluate $\frac{3a+18b-c}{-2a}$ when $a = -2$; $b = 1$ and $c = -8$**
   - **a)** 1
   - **b)** −1
   - **c)** 5
   - **d)** 4

2. Solving linear equations

A **linear equation** is an equation where the highest exponent on any unknown variable is 1.

To **solve** a linear equation means to find all values of the unknown variable which will make the equation true (i.e. left side of the equation *equals* the right side of the equation).

A linear equation with **one unknown variable** usually has **one solution.**

Note: Equations with no solution and equations where the solution is all real numbers are not discussed here.

> To solve a linear equation with one unknown variable**, isolate the variable to one side of the equation** by **moving all the other terms to the other side of the equation** and **simplifying**. To move a term to the other side of the equation, do the opposite operation.

Opposite operations	
addition	subtraction
multiplication	division
exponent of 2 (squared)	square root ($\sqrt{\ }$)
any positive exponent “n”	“nth” root ($\sqrt[n]{\ }$)

A **LS/RS** (left side/right side) **Check** can be performed to check the solution. To do a LS/RS Check, **substitute your solution into the original equation**. Evaluate the LS and the RS of the equation independently. If LS = RS than your solution is correct. If the LS does NOT equal the RS than your solution is incorrect.

Example 1:

Solve for x.

3x + 18 = 3

3x + 18 = 3 — Step 1: Move 18 to the other side of the equation by subtracting 18 on the right side.

3x = 3 – 18 — Simplify the right side.

3x = − 15 — Step 2: Move 3 to the other side of the equation by dividing by 3 on the right side of the equation.

x = $\frac{-15}{3}$ — Simplify the right side.

x = − 5 — Solution to the equation is – 5.

Do a LS/RS Check to check the solution.

LS	RS
3x + 18 = 3(−5) + 18 = −15 + 18 = 3	3

LS = RS

Example 2:

Solve for x.

$$\frac{20x+4}{9} = 4$$

$$\frac{20x+4}{9} = 4$$

Step 1: Move 9 to the right side of the equation by multiplying by 9 on the right side.

$20x + 4 = 4(9)$

Simplify the right side.

$20x + 4 = 36$

$20x + 4 = 36$

Step 2: Move 4 to the right side of the equation by subtracting 4 on the right side.

$20x = 36 - 4$

Simplify the right side.

$20x = 33$

$20x = 32$

Step 3: Move 20 to the right side of the equation by dividing by 20 on the right side.

$x = \frac{32}{20}$

Simplify the right side.

$x = 1.6$ OR $x = 1\frac{6}{10}$

Do a LS/RS Check to make sure the solution is correct.

LS	RS
$\frac{20x+4}{9}$	$= 4$
$= \frac{20(1.6)+4}{9}$	
$= \frac{32+4}{9}$	
$= \frac{36}{9}$	
$= 4$	

LS = RS

Practice questions:

3. Solve for x.
$5x + 14 = -6$

a) -4
b) 4
c) -100
d) -15

4. Solve for x.
$$\frac{-3x+2}{6} = \frac{5}{6}$$

a) 2
b) 1
c) -1
d) $\frac{1}{2}$

3. Addition and subtraction of polynomials

Addition and subtraction of monomials

Two or more monomials can be added or subtracted only if they are **like terms.**

Like terms are terms that have exactly the **SAME variables and exponents** on those variables. The coefficients on like terms may be different.

Example:

$7x^2y^5$ and $-2x^2y^5$ These are like terms since both terms have the same variables and the same exponents on those variables.

$7x^2y^5$ and $-2x^3y^5$ These are NOT like terms since the exponents on x are different.

Note: the **order** that the variables are written in does NOT matter. The different variables and the coefficient in a term are ***multiplied*** *together* and the order of multiplication does NOT matter (For example, 2 x 3 gives the same product as 3 x 2).

Example:

$8a^3bc^5$ is the *same* term as $8c^5a^3b$.

To prove this, evaluate both terms when a = 2, b = 3 and c = 1.

$8a^3bc^5 = 8(2)^3(3)(1)^5$
$= 8(8)(3)(1)$
$= 192$

$8c^5a^3b = 8(1)^5(2)^3(3)$
$= 8(1)(8)(3)$
$= 192$

As shown, both terms are equal to 192.

Practice question:

5. Which of the following are like terms with $6x^5y$?
- **a)** $3x^5$
- **b)** $-yx^5$
- **c)** $-2x^4y$
- **d)** $7y^2$

To add two or more monomials that are like terms, add the coefficients; keep the variables and exponents on the variables the same.

To subtract two or more monomials that are like terms, subtract the coefficients; keep the variables and exponents on the variables the same.

Example 1:
Add $9xy^2$ and $-8xy^2$

$9xy^2 + (-8xy^2) = [9 + (-8)]\, xy^2$
$= 1xy^2$
$= xy^2$

Add the coefficients. Keep the variables and exponents on the variables the same.

Note: By convention, a coefficient of 1 does not have to be explicitly written. If there is no coefficient on a term, it is assumed to be a coefficient of 1. Likewise, if there is no exponent on a variable in a term, it is assumed to be an exponent of 1.

Example 2:

Subtract. $10y^2 - (-xy^2) - 17y^2 - xy^2$

$10y^2 - (-xy^2) - 17y^2 - xy^2$

Step 1: Only like terms can be subtracted. In this algebraic expression, like terms are $10y^2$ and $17y^2$ and $-xy^2$ and xy^2.

$= 10y^2 - 17y^2 - (-1xy^2) - 1xy^2$ Step 2: Subtract the coefficients of like terms.

$= -7y^2 + 0xy^2$ Step 3: Simplify $0xy^2$ to 0 since 0 multiplied by anything equals 0.

$= -7y^2 + 0$ Step 4: $-7y^2$ plus 0 is just $-7y^2$

$= -7y^2$

Practice questions:

6. **Find the sum of $3d^5c$, $-12cd^5$, $8d^5$ and $5c$.**
   a) $4d^5c$
   b) $-d^5c + 5c$
   c) $-4cd^5 + 8d^5$
   d) $-9cd^5 + 8d^5 + 5c$

7. **Simplify.**
   $24x^3 - 18k^2v - (-24x^3) - 6k^2v$
   a) $-24k^2v$
   b) $48x^3 - 24k^2v$
   c) $48x^3$
   d) $-12k^2v - 48x^3$

8. **Simplify.**
   $12d + (-3x^3y) - 5d + 7x^3 - x^3y$
   a) $7d + 2x^3y$
   b) $14dx^3 - 4x^3y$
   c) $7d - 4x^3y + 7x^3$
   d) $10dx^3y$

4. <u>Multiplication of polynomials</u>

<u>The product of a monomial x monomial</u>

To multiply a monomial times a monomial, **multiply the coefficients** and **add the exponents on powers with the same variable as a base**.

Note: A common mistake that many students make is to multiply the exponents on powers with the same variables as a base. This is NOT CORRECT. Remember the exponent rules!

Exponent Rules			
Case	What to do	Rule	Example
Multiplying powers with the same base	Add the exponents. Keep the base the same.	$(x^a)(x^b) = x^{a+b}$	$(2^5)(2^3) = 2^8$
Dividing powers with the same base	Subtract the exponents. Keep the base the same.	$\frac{x^a}{x^b} = x^a \div x^b = x^{a-b}$	$\frac{2^5}{2^3} = 2^5 \div 2^3 = 2^2$
Simplifying power of a power	Multiply the exponents. Keep the base the same.	$(x^a)^b = x^{ab}$	$(2^5)^3 = 2^{15}$
Exponent of 0	Anything to the exponent of 0 equals 1.	$x^0 = 1$	$2^0 = 1$

Other cases that come up when working with powers		
Case	What to do	Example
Adding powers with the same base and SAME exponents	The powers are like terms. Add the coefficients; keep the base and the exponent the same.	$x^a + x^a = 2x^a$
Adding powers with the same base and DIFFERENT exponents	The powers are NOT like terms. They can NOT be added.	$x^a + x^b = x^a + x^b$
Subtracting powers with the same base and SAME exponents	The powers are like terms. Subtract the coefficients; keep the base and the exponent the same.	$2x^a - x^a = x^a$
Subtracting powers with the same base and DIFFERENT exponents	The powers are NOT like terms. They can NOT be subtracted.	$x^a - x^b = x^a - x^b$

Example 1:

Simplify. $5x^3(-6x)$

$= 5(-6)x^{3+1}$ Multiply the coefficients. Add the exponents since both powers have base x.

$= -30x^4$

Example 2:

Simplify. $20a^2y^{-3}b\,(\frac{1}{4}ay^4)$

$= 20(\frac{1}{4})a^{2+1}y^{-3+4}b$ — Multiply the coefficients. Add the exponents for powers with base a. Add the exponents for powers with base y.

$= 5a^3yb$

Example 3:

Simplify. $0.10p^{10}q^4\,(10)p^5q^{-4}$

$0.10p^{10}q^4\,(10)p^5q^{-4}$

$= 0.10(10)p^{10+5}q^{4-4}$ — Multiply the coefficients. Add the exponents for powers with base a. Add the exponents for powers with base y.

$= 1p^{15}q^0$ — Simplify q^0. Any number to the exponent of 0 equals 1.

$= p^{15}(1)$ — p^{15} multiplied by 1 is just p^{15}

$= p^{15}$

Practice questions:

9. **Simplify.**
   $18x^5y^8\,(3x^2y^5)$

   a) $6x^3y^3$
   b) $54x^7y^8$
   c) $54x^{10}y^{40}$
   d) $54x^7y^{13}$

10. **Simplify.**
    $-8k^{-2}g(-3k^2p)(2k^1g^9)$

    a) $48pg^{10}$
    b) $48kpg^{10}$
    c) $24kpg^{10}$
    d) $-48kpg^{10}$

<u>The product of a monomial x binomial</u>

To multiply a monomial by a binomial, **multiply the monomial by EVERY term making up the binomial.**

Remember: To find the product of two terms, multiply the coefficients and **add the exponents on powers with the same variable as a base**.

Example 1:

Expand. $5x^3(7x^2 + 15xy)$

$5x^3(7x^2 + 15xy)$ Step 1: Multiply the monomial by EVERY term making up the binomial.

$= 5x^3(7x^2) + 5x^3(15xy)$ Step 2: To find the product of two terms, multiply the coefficients and add the exponents on powers with the same variable as a base.

$= 35x^5 + 75x^4y$

Example 2:

Expand. $-2c^4p(10c^3p^2 - 4c)$

$-2c^4p(10c^3p^2 - 4c)$ Step 1: Multiply the monomial by EVERY term making up the binomial.

$= -2c^4p(10c^3p^2) + (-2c^4p)(-4c)$ Step 2: To find the product of two terms, multiply the coefficients and add the exponents on powers with the same variable as a base.

$= -20c^7p^3 + 8c^5p$

Practice question:

11. Expand.
$x^2y(-2xy + 12x^{-2}y)$

- **a)** $-2x^3y^2 + 12y^2$
- **b)** $-2x^3y^2 + 6xy$
- **c)** $-x^3y^2 + 6y^2$
- **d)** $-2x^3y + 6y^2$

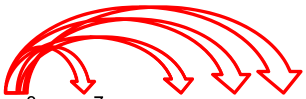
The product of a monomial x trinomial OR monomial x polynomial

To **multiply a monomial by a trinomial or any polynomial, multiply EVERY term in the trinomial or polynomial by the monomial**.

To find the product of two terms, multiply the coefficients and add the exponents on powers with the same variable as a base.

Example:

Expand. $7x^3(19x^7y + 20 - 3x + y)$



$7x^3(19x^7y + 20 - 3x + y)$

Step 1: Multiply the monomial by EVERY term making up the binomial.

$= 7x^3(19x^7y) + 7x^3\,(20) + 7x^3\,(-3x) + 7x^3\,(y)$

Step 2: To find the product of two terms, multiply the coefficients and add the exponents on powers with the same variable as a base.

$= 133x^{10}y + 140x^3 - 21x^4 + 7x^3y$

Practice question:

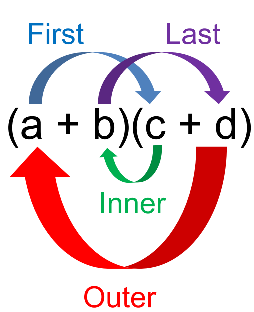
12. Expand.
$y^3x^2\,(4y^2x + 6x - 12)$

a) $4y^3x + 24y^3x^3 - 12$
b) $4y^5x + 6y^3x^3 - 12$
c) $4y^5x + 6y^3x^3 + 12y^3$
d) $4y^5x^3 + 6y^3x^3 - 12y^3x^2$

The product of a binomial x binomial

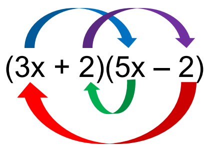
To multiply a binomial by a binomial, multiply EVERY term in the first binomial by EVERY term in the second binomial. Then simplify by collecting (adding or subtracting) like terms, if it is possible.

You can use the **FOIL** (First, Outer, Inner, Last) **method** to remember how to multiply binomials.



$= ac + ad + bc + bd$

Example 1:
Expand. (3x + 2)(5x – 2)



Step 1: Use FOIL to multiply every term in the first binomial by every term in the second binomial.

$= 3x(5x) + (3x)(-2) + (2)(5x) + (2)(-2)$

Step 2: Evaluate every product.

$= 15x^2 + (-6x) + 10x + (-4)$

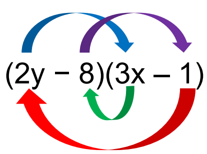
$= 15x^2 - 6x + 10x - 4$

Step 3: Collect like terms.

$= 15x^2 + 4x - 4$

This is the final answer.

Example 2:
Expand. (2y − 8)(3x – 1)



Step 1: Use FOIL to multiply every term in the first binomial by every term in the second binomial.

$= 2y(3x) + (2y)(-1) + (-8)(3x) + (-8)(-1)$

Step 2: Evaluate every product.

$= 6yx + (-2y) + (-24x) + (8)$

There are no like terms that can be collected. Simplify double signs. Arrange terms in alphabetical order*.

$= 6yx - 24x - 2y + 8$

This is the final answer.

*Note: By convention, terms are written from *highest* to *lowest degree* and in alphabetical order. (See Glossary).

Practice questions:

13. Expand. (9x + 7)(3x – 1)

- **a)** $27x^2 - 7$
- **b)** $21x - 9x$
- **c)** $12x^2 + 30x - 7$
- **d)** $27x^2 + 12x - 7$

14. Expand. (5y + 3)(5y – 3)

- **a)** $25y^2 - 9$
- **b)** $25y^2 + 30y - 9$

c) $10y - 6$
d) $25y^2 + 9$

Squaring a binomial

To square a binomial means to multiply the binomial by itself.

The rules of multiplying a binomial by a binomial apply. To multiply a binomial by a binomial, multiply EVERY term in the first binomial by EVERY term in the second binomial.

When multiplying a binomial by itself, the expanding follows a pattern as shown below.

$$(a + b)^2 = (a + b)(a + b) = a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$$

Example 1:

Expand. $(5x - 3)^2$

Solution 1:

$(5x - 3)^2 = (5x - 3)(5x - 3)$ — Step 1: Use FOIL method to expand.

$= 25x^2 - 15x - 15x + 9$ — Step 2: Collect like terms.

$= 25x^2 - 30x + 9$

Solution 2:

$(5x - 3)^2 = (5x)^2 + 2(5x)(-3) + (-3)^2$ — Step 1: Use the $(a + b)^2 = a^2 + 2ab + b^2$ pattern to expand.

$(a + b)^2 = a^2 + 2ab + b^2$

$= 25x^2 - 30x + 9$ — Step 2: Simplify each term.

Example 2:

Expand. $(9y + 2)^2$

$(9y + 2)^2 = (9y)^2 + 2(9y)(2) + 2^2$ — Step 1: Use the $(a + b)^2 = a^2 + 2ab + b^2$ pattern to expand

$= 81y^2 + 36y + 4$ — Step 2: Simplify each term.

Practice questions:

15. Expand. $(9y – 8)^2$

- **a)** $9y^2 + 64$
- **b)** $81y^2 + 64$
- **c)** $81y^2 – 144y + 64$
- **d)** $81y^2 + 144y – 64$

16. Expand. $(2x + 3y)^2$

- **a)** $4x^2 + 9y^2$
- **b)** $4x^2 + 6xy + 9y^2$
- **c)** $4x^2 + 12xy + 9y^2$
- **d)** $2x^2 + 12xy + 3y^2$

5. Factoring

Factoring is the OPPOSITE operation of expanding algebraic expressions.

To factor an algebraic expression means to find the numbers, monomials, binomials or polynomials that multiplied together result in the given algebraic expression.

This study guide will focus on common factoring algebraic expressions; factoring trinomials of the form $ax^2 + bx + c$ and two special cases of factoring: difference of squares and perfect square trinomial.

Common factoring

Step 1: Find the **greatest common factor of all terms** in the algebraic expression. Consider the numbers and variables making up each term.

Step 2: Write the common factor in front of the brackets. In brackets, write the algebraic expression resulting from dividing EACH term by the common factor.

Example 1:

Factor $16xy^2 + 20x^2y – 4x^3y^2$.

Step 1: Find the greatest common factor of $16xy^2$, $20x^2y$ and $4x^3y^2$.

Look at the numbers 16, 20 and 4. The greatest common factor of these numbers is 4, since all of these numbers can be divided by 4 evenly.

$16 \div 4 = 4$

$20 \div 4 = 5$

$4 \div 4 = 1$

Look at the variables in each term, xy^2, x^2y and x^3y^2. The greatest common factor of these variables is xy since all of these terms can be divided by xy evenly.

$xy^2 \div xy = y$

$x^2y \div xy = x$

$x^3y^2 \div xy = x^2y$

(Note: the terms are divided according to the exponent rules. See page 8.)

Therefore, the greatest common factor is 4xy.

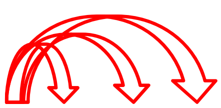
<u>Step 2: Write the greatest common factor in front of brackets. Determine the algebraic expression in brackets by dividing each term in the given algebraic expression by the greatest common factor.</u>

$$4xy\left(\frac{16xy^2}{4xy} + \frac{20x^2y}{4xy} - \frac{4x^3y^2}{4xy}\right)$$

$$= 4xy(4y + 5x - x^2y)$$

Notice that the terms in the bracket are the same numbers and variables seen above in Step 1, when we divided numbers and variables by the greatest common factors.

<u>Step 3 (Optional): Double check the answer by expanding $4xy(4y + 5x - x^2y)$. The answer should be the algebraic expression that was given in the question.</u>



$4xy(4y + 5x - x^2y)$

$= 4xy(4y) + 4xy(5x) + 4xy(-x^2y)$

$= 16xy^2 + 20x^2y - 4x^3y^2$

Example 2:

Factor $-144p^5q - 12p^3 - 6p$.

<u>Step 1:</u> Find the greatest common factor of $-144p^5q$, $-12p^3$ and $-6p$.

The greatest common factor of –144, –12 and −6 is −6.

$-144 \div (-6) = 24$

$-12 \div (-6) = 2$

$-6 \div (-6) = 1$

The greatest common factor of p^5q, p^3 and p is p.

$p^5q \div p = p^4q$

$p^3 \div p = p^2$

$p \div p = 1$

Therefore, the greatest common factor is −6p.

Step 2: Write the greatest common factor in front of brackets. Determine the algebraic expression in brackets by dividing each term in the given algebraic expression by the greatest common factor.

$-6p\left(\frac{-144p^5q}{-6p} - \frac{12p^3}{-6p} - \frac{6p}{-6p}\right)$

$= -6p\,(24p^4q + 2p^2 + 1)$

Practice questions:

17. Factor.

$24k^3s^2 + 12k^2s^5 - 48k^2s^3$

a) $12k^2s^2\,(2k + s^3 - 4s)$
b) $2ks^2\,(12k^2 + 6ks^3 - 24k^2s)$
c) $12\,(2k^3s^2 + k^2s^4 - 4k^2s^3)$
d) $12k^2s^2\,(2k + s^5 - 4s)$

18. Factor.

$12x^3 + 6x - 2y$

a) $2x(6x^2 + 3x - y)$
b) $2x(6x^2 + 3 - xy)$
c) $2(6x^3 - 3x + y)$
d) $2(6x^3 + 3x - y)$

Factoring the trinomial $ax^2 + bx + c$ when a = 1

A trinomial in the form **$x^2 + bx + c$** can be factored to equal **(x + m)(x + n)** when the product of m x n equals c and the sum of m + n equals b.
(Note: the coefficient in front of x^2 must be 1)

Step 1: Common factor if you can.

Step 2: Find two integers (negative or positive whole numbers), m and n, that multiply to equal c (from $x^2 + bx + c$) AND add to equal b (from $x^2 + bx + c$).

Start with finding integers that give you a product c and then check which pair of numbers will add to equal b. Start with the product since there is a limited number of pairs that will give you the product c, while there is an infinite amount of numbers that can add to equal b.

Step 3: Substitute the numbers m and n directly into the expression (x + m)(x + n)

Example 1:

Factor. $x^2 + 7x + 12$

<u>Step 1:</u> Check to see if you can common factor first. In this case, there are <u>no</u> common factors for x^2, $7x$ and 12.

<u>Step 2:</u> Find two integers (negative or positive whole numbers), m and n, that multiply to equal 12 AND add to equal 7.

$x^2 + \mathbf{7}x + \mathbf{12}$

$x^2 + \mathbf{b}x + \mathbf{c}$

$b = 7$

$c = 12$

12 = 12 x 1	12 = (−12)(−1)
12 = 6 x 2	12 = (−6)(−2)
12 = 4 x 3	12 = (−4)(−3)

From the above pairs of integers only 3 and 4 add to 7.

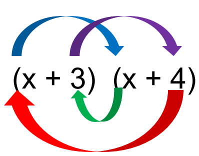
<u>Step 3:</u> Substitute 3 and 4 into (x + m)(x + n)

$(x + \mathbf{m})(x + \mathbf{n})$

$(x + \mathbf{3})(x + \mathbf{4})$

Thus, $x^2 + 7x + 12 = (x + 3)(x + 4)$

<u>Step 3:</u> To double check that the factoring was done correctly, expand (x + 3)(x + 4).



$= x^2 + 4x + 3x + 12$

$= x^2 + 7x + 12$

Example 2:

Factor. $2x^2 - 30x + 72$

<u>Step 1:</u> Common factor first, since $2x^2$, $-30x$ and 72 are all divisible by 2.

$2x^2 - 30x + 72 = 2(x^2 - 15x + 36)$

<u>Step 2:</u> Now look at $x^2 - 15x + 36$ only and factor it.

$x^2 \mathbf{-15}x + 36$

$x^2 + \mathbf{b}x + \mathbf{c}$

b = −15

c = 36

Find two integers (negative or positive whole numbers), m and n, that multiply to equal 36 AND add to equal −15.

36 = 36 x 1	36 = (−36)(−1)
36 = 18 x 2	36 = (−18)(−2)
36 = 12 x 3	36 = (−12)(−3)
36 = 9 x 4	36 = (−9)(−4)
36 = 6 x 6	36 = (−6)(−6)

From the above pairs of integers only −12 and −3 add to −15.

Step 3: Substitute −12 and −3 into (x + m)(x + n)

(x + m)(x + n) = [x + (−12)] [x + (−3)]

= (x −12)(x − 3)

Step 4: Put all of the factors together. Remember the common factor from Step 1.

x^2 −15x + 36 = 2(x −12)(x − 3)

Practice Questions:

19. Factor.

$x^2 – 4x + 3$

- **a)** (x + 3)(x – 3)
- **b)** (x – 3)(x – 1)
- **c)** (x + 2)(x – 1)
- **d)** (x + 3)(x + 1)

20. Factor.

$5y^2 + 20y + 20$

- **a)** (5y + 4)(y + 5)
- **b)** 5(y + 4)(y + 1)
- **c)** 5(y + 2)(y + 2)
- **d)** (5y + 2)(y + 2)

Factoring the trinomial $ax^2 + bx + c$ when $a \neq 1$

The ***decomposition method*** to factor a trinomial in the form $ax^2 + bx + c$ when a does not equal 1 is presented on the next page.

Step 1: Check to see if you can common factor first.

Decomposition Method:

Step 2: Find two integers such that the **product of these integers equals the product of a and c, ac,** (from $ax^2 + bx + c$) AND the **sum of these integers equals b** (from $ax^2 + bx + c$).

Step 3: Use the two integers from Step 2 to re-write the middle term, bx, as the sum of these two integers.

Step 4: Common factor the first **two** terms of the algebraic expression. Then, common factor the last **two** terms of the algebraic expression. The objective of this step is to get two factors or brackets that are the same.

Step 5: Common factor the whole algebraic expression from Step 4.

Example 1:

Factor. $6x^2 + 13x - 5$.

Step 1: There is no common factor for 6, 13 and −5.

Step 2: Find two integers such that the product of these integers equals ac and the sum equals b.

$6x^2 + 13x - 5$

$ax^2 + bx + c$

$ac = 6(-5) = -30$

$b = 13$

Start with looking for two integers whose product is −30 since there is an infinite number of pairs of integers whose sum is 13.

$5(-6) = -30$	$-5(6) = -30$
$3(-10) = 30$	$-3(10) = 30$
$15(-2) = -30$	$-15(2) = -30$
$1(-30) = -30$	$-1(30) = -30$

From the above pairs only $15 + (-2) = 13$.

Step 3: Use 15 and −2 to re-write the middle term, 13x, as the sum of these two integers.

$13x = 15x + (-2x)$

Therefore, $6x^2 + 13x - 5 = 6x^2 + 15x - 2x - 5$

Step 4: Common factor the first two terms of the algebraic expression, $6x^2 + 15x$. Then, common factor the last two terms of the algebraic expression, $-2x - 5$.

$6x^2 + 15x - 2x - 5$

$3x(2x + 5)$ **$-1(2x + 5)$**

Note: Factor out −1, so the terms in the two brackets match.

<u>Step 5:</u> Common factor the resulting algebraic expression.

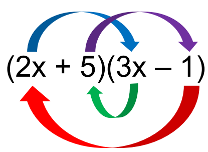
$3x$**$(2x + 5)$** -1**$(2x + 5)$** The common factor here is $(2x + 5)$ since both terms, $3x(2x+5)$ and $-1(2x+5)$ are divisible by $2x + 5$.

$= (2x + 5)\left(\frac{3x}{2x+5} - \frac{1}{2x+5}\right)$

$= (2x + 5)(3x - 1)$

Therefore, $6x^2 + 13x - 5 = (2x + 5)(3x - 1)$

<u>Step 6 (Optional):</u> To check the answer, expand the factored expression.



$= 6x^2 - 2x + 15x - 5$

$= 6x^2 + 13x - 5$

Example 2:

Factor. $-14x^2 + 116x - 32$.

<u>Step 1:</u> This trinomial can be common factored since −14, 116 and – 32 are all divisible by −2.

$-14x^2 + 116x - 32 = -2(7x^2 - 58x + 16)$

<u>Step 2:</u> Use decomposition method to factor the $7x^2 - 58x + 16$ trinomial.

$ac = 7(16) = 112$

$b = -58$

Note: Since the product of the two integers is *positive*, but the sum is *negative*, **both** integers MUST be negative.

$-4(-28) = 112$

$-2(-56) = 112$

$(-14)(-8) = 112$

$(-7)(-16) = 112$

From the above list, only −2+(−56) = −58

Step 3: Use −2 and −56 to re-write the middle term, −58x, as the sum of these two integers.

$-58x = -2x + (-56x) = -2x - 56x$

Therefore, $7x^2 - 58x + 16 = 7x^2 - 2x - 56x + 16$

Step 4: Common factor the first two terms of the algebraic expression, $7x^2 - 2x$. Then, common factor the last two terms of the algebraic expression, $-56x + 16$.

$7x^2 - 2x - 56x + 16$

$= x(7x - 2) - 8(7x - 2)$

Factor out −8, not 8, since we want the brackets to be the same so that they become a common factor.

Step 5: Common factor the resulting algebraic expression.

$x(7x - 2) - 8(7x - 2) = (7x - 2)(x - 8)$

Step 6 (Optional): To check the answer, expand the factored expression.

$(7x - 2)(x - 8)$

$= 7x^2 - 56x - 2x + 16$

$= 7x^2 - 58x + 16$

Practice Questions:

21. Factor.

$54x^2 + 12x - 10$

- **a)** $4(2x + 1)(9x + 5)$
- **b)** $(6x + 2)(9x + 5)$
- **c)** $2(3x - 1)(9x + 5)$
- **d)** $4(3x - 1)(9x + 5)$

22. Factor.

$33y^2 + 5y - 2$

- **a)** $(11y - 2)(3y + 1)$
- **b)** $(11y + 2)(3y - 1)$
- **c)** $(11y + 1)(3y - 2)$
- **d)** $(11y - 1)(3y + 2)$

Special case: Difference of squares

Difference of squares is a special case of factoring, which follows a specific pattern. Firstly, it is important to be able to recognize a difference of squares.

For an algebraic expression to be a difference of squares **the first and last terms must be perfect squares. The two perfect squares must be subtracted.**

Perfect square – a number whose square root is a whole number and/or a variable with an even exponent.

Numbers that are perfect squares: 1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, …

Variables are perfect squares if they have an ***even*** exponent (e.g. 2, 4, 6, 8, etc.)

Examples of Difference of Squares			NOT Examples of Difference of Squares		
$4x^2 - 9$	$16y^4 - 25$	$169p^8 - 1$	$4x^2 + 9$	$16y^5 - 25$	$168p^8 - 1$

Practice Question:

23. Which of the following algebraic expressions is a difference of squares?

a) $8x^2 - 25$
b) $9x^3 - 25$
c) $9x^2 + 25$
d) $9x^2 - 25$

If you can recognize a difference of squares, the factoring can be done in one line according to the pattern below.

Factoring a difference of squares:

$$a^2 - c^2 = (a + c)(a - c)$$

Notice that the first term inside both brackets is the square root of a^2; the second term inside both brackets is the square root of c^2; the operation sign inside one bracket is + and inside the other bracket is −.

Example:

Factor. $25x^2 - 121$.

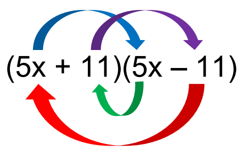
Step 1: Try to common factor first. There are no common factors of $25x^2$ and 121.

Step 2: Recognize that this is a difference of squares since $25x^2$ and 121 are both perfect squares AND these two perfect squares are being subtracted.

Step 3: Factor according to $a^2 - c^2 = (a + c)(a - c)$.

$$25x^2 - 121 = (\sqrt{25x^2} + \sqrt{121})(\sqrt{25x^2} - \sqrt{121})$$
$$= (5x + 11)(5x - 11)$$

Step 4 (Optional): To double check the answer, expand $(5x + 11)(5x - 11)$. The result of expanding should be $25x^2 - 121$.



$= 25x^2 - 55x + 55x - 121$

$= 25x^2 - 121 + 0x$

$= 25x^2 - 121$

> A difference of squares can sometimes be "disguised" when it is multiplied by a common factor. This is why it's important to **always try to common factor an algebraic expression first**.

Example:

Factor. $98x^3 - 2x$.

Step 1: Common factor the expression first.

98 and 2 are both divisible by 2

x^3 and x are both divisible by x

Thus, the common factor of $98x^3$ and $2x$ is $2x$.

$98x^3 - 2x = 2x(49x^2 - 1)$

Notice that the algebraic expression inside the brackets, $49x^2 - 1$, is a difference of squares.

Step 2: Factor the difference of squares according to the pattern.

$$49x^2 - 1 = (\sqrt{49x^2} + \sqrt{1})(\sqrt{49x^2} - \sqrt{1})$$
$$= (7x + 1)(7x - 1)$$

Step 3: Write the final answer by putting all of the factors together.

$98x^3 - 2x = 2x(7x + 1)(7x - 1)$

Step 4 (Optional): To double check the answer, expand $2x(7x + 1)(7x - 1)$. The result of expanding should be $98x^3 - 2x$.

$2x(7x + 1)(7x - 1)$	Multiply $2x$ by each term in the first bracket, $7x + 1$. (Note: $2x$ does NOT get multiplied by *each* bracket since you would be multiplying $2x$ twice).
$= (14x^2 + 2x)(7x - 1)$	Use FOIL method to multiply binomial by a binomial.
$= 98x^3 - 2x + 14x^2 - 14x^2$	Collect like terms.
$= 98x^3 - 2x + 0x^2$	Simplify $0x^2$.
$= 98x^3 - 2x$	

Practice Questions:

24. Factor.

$144y^2 - 49$

- **a)** $(12y - 7)^2$
- **b)** $(72y + 7)^2$
- **c)** $(72y - 7)(12y + 7)$
- **d)** $(12y + 7)(12y - 7)$

25. Factor.

$64x^4 - 100x^2$

- **a)** $x^2(4x + 10)(4x - 10)$
- **b)** $4x^2(4x + 5)^2$
- **c)** $4x^2(4x + 5)(4x - 5)$
- **d)** $(8x^2 + 10)(8x^2 - 10)$

Special case: Perfect square trinomial

Perfect square trinomial is another special case of factoring, which follows a specific pattern. Firstly, it is important to be able to recognize a perfect square trinomial.

> For an algebraic expression to be a perfect square trinomial **the first and last terms must be perfect squares. The middle term has to equal to twice the square root of the first term times the square root of the last term.**

Examples of Perfect Square Trinomials	NOT Examples of Perfect Square Trinomials	
$x^2 + 12x + 36$	$x^2 + 12x - 36$	-36 is NOT a perfect square
$16y^2 - 40y + 25$	$16y^2 - 41y + 25$	$-41y$ does NOT equal to $2(\sqrt{16y^2})(\sqrt{25})$
$169p^2 + 26p - 1$	$168p^2 + 26p - 1$	$168p^2$ is NOT a perfect square
$4x^2 + 12x + 9$	$4x^2 + 12x + 8$	8 is NOT a perfect square

Practice Question:

26. Which of the following algebraic expressions is a perfect square trinomial?

a) $25x^2 + 30x + 9$
b) $15x^2 + 10x + 1$
c) $36x^3 - 12x + 1$
d) $25x^2 + 30x - 9$

If you can recognize a perfect square trinomial, the factoring can be done in one line according to the pattern below.

Factoring a Perfect Square Trinomial:

$$a^2 + 2ac + c^2 = (a + c)(a + c) = (a + c)^2$$

$$a^2 - 2ac + c^2 = (a - c)(a - c) = (a - c)^2$$

Notice that the first term inside both brackets is the square root of a^2; the second term inside both brackets is the square root of c^2; the operation sign (+ or −) inside the brackets is the same as the operation sign in front of 2ac.

Example:

Factor. $36x^2 - 132x + 121$.

Step 1: Try to common factor first. There are no common factors of $36x^2$, 132x and 121.

Step 2: Recognize that this is a perfect square trinomial since $36x^2$ and 121 are both perfect squares AND the middle term, 132x, is equal to $2(\sqrt{36x^2})(\sqrt{121}) = 2(6x)(11)$

Step 3: Factor according to $a^2 - 2ac + c^2 = (a - c)^2$

$$36x^2 - 132x + 121 = (\sqrt{36x^2} - \sqrt{121})^2 = (6x - 11)^2$$

$$a^2 - 2ac + c^2 = (\sqrt{a^2} - \sqrt{c^2})^2 = (a - c)^2$$

Step 4 (Optional): To double check the answer, expand $(6x - 11)^2$. The result of expanding should be $36x^2 - 132x + 121$.

$$(6x - 11)^2 = (6x - 11)(6x - 11)$$

$$= 36x^2 - 66x - 66x + 121$$

$$= 36x^2 - 132x + 121$$

A perfect square trinomial can sometimes be "disguised" when it is multiplied by a common factor. This is why it's important to **always try to common factor an algebraic expression first**.

Example:

Factor. $-18x^3 - 96x^2 - 128x$

Step 1: Common factor the expression first.

$-18x^3$, $-96x^2$ and $-128x$ are all divisible by $-2x$.

Thus, the common factor is $-2x$.

$-18x^3 - 96x^2 - 128x = -2x(9x^2 + 48x + 64)$

Notice that the algebraic expression inside the brackets, $9x^2 + 48x + 64$, is a perfect square trinomial.

Step 2: Factor the perfect square trinomial according to $a^2 + 2ac + c^2 = (a + c)^2$

$$9x^2 + 48x + 64 = (\sqrt{9x^2} + \sqrt{64})^2 = (3x + 8)^2$$

$$a^2 + 2ac + c^2 = (\sqrt{a^2} + \sqrt{c^2})^2 = (a + c)^2$$

Step 3: Write the final answer by putting all the factors together.

$-18x^3 - 96x^2 - 128x = -2x(3x + 8)^2$

From Step 1. From Step 2.

Step 4 (Optional): To double check the answer, expand $-2x(3x + 8)^2$. The result of expanding should be $-18x^3 - 96x^2 - 128x$.

$$-2x(3x + 8)^2 = -2x(3x + 8)(3x + 8)$$

$$= (-6x^2 - 16x)(3x + 8)$$

$$= -18x^3 - 48x^2 - 48x^2 - 128x$$

$$= -18x^3 - 96x^2 - 128x$$

Practice Questions:

27. Factor.

$169y^2 - 182y + 49$

a) $(13y - 7)(13y + 7)$
b) $(13y - 7)^2$
c) $(13y + 7)^2$
d) $y(13y + 7)$

28. Factor.

$192 + 3y^2 + 48y$

a) $3y(y + 8)$
b) $(8 + y)^2$
c) $3(y - 8)^2$
d) $3(y + 8)(y + 8)$

29. Factor.

$2y^4 - 2$

a) $(2y^2 + 1)(2y^2 - 1)$
b) $(2y^2 + 1)(y + 1)(y - 1)$
c) $2(y^2 + 1)(y + 1)(y - 1)$
d) $2(y^2 + 1)^2$

6. Operations with algebraic fractions

Operations with algebraic fractions follow the same rules as operations with fractions:

1. Algebraic fractions can be **added or subtracted** ONLY if they have the **SAME DENOMINATOR** (a common denominator).
   To find a common denominator, find the **least common multiple** of the denominators of all algebraic fractions being added or subtracted.

2. When **multiplying** algebraic fractions, **multiply the numerator by the numerator** and **denominator by denominator**.

3. When **dividing** algebraic fractions, **multiply by the reciprocal**. The **reciprocal** is the multiplicative inverse of a number. For a fraction $\frac{a}{b}$, the reciprocal is $\frac{b}{a}$.

IMPORTANT: For algebraic fractions containing **trinomials** in the form $\mathbf{ax^2 + bx + c}$, try to **factor** the trinomials. When the trinomials are factored it is much easier to see what the common denominator is. Furthermore, some of the factors may cancel out if they appear both in the numerator and in the denominator of the answer.

Example 1:

Simplify.

$$\frac{b(b-4)}{b^2} + \frac{2b}{a}$$

Step 1: To add the algebraic fractions, find a common denominator.

The least common multiple of $\mathbf{b^2}$ and **a** is $\mathbf{b^2a}$. Thus, the common denominator is b^2a.

Multiply the numerator AND the denominator of the first algebraic fraction, $\frac{b(b-4)}{b^2}$ by **a**. Simplify.

Multiply the numerator AND the denominator of the second algebraic fraction , $\frac{2b}{a}$ by $\mathbf{b^2}$. Simplify.

$$\frac{[b(b-4)](a)}{b^2(a)} + \frac{2b(b^2)}{a(b^2)}$$

$$= \frac{ab(b-4)}{ab^2} + \frac{2b^3}{ab^2}$$

Step 2: Add the numerators of the algebraic fractions. Simplify.

Keep the denominator the same.

$$\frac{ab(b-4) + 2b^3}{ab^2}$$

$$= \frac{ab^2 - 4ab + 2b^3}{ab^2}$$

$$= \frac{2b^3 + ab^2 - 4ab}{ab^2}$$

Example 2:
Simplify.

$$\frac{5x - 3}{2x + 9} - \frac{1}{x + 8}$$

Step 1: To subtract algebraic fractions, find a common denominator.

The common denominator is $(2x + 9)(x + 8)$.

Multiply the numerator AND denominator of the first algebraic fraction by $(x + 8)$.

Multiply the numerator AND denominator of the second algebraic fraction by $(2x + 9)$.

$$\frac{(5x - 3)(x + 8)}{(2x + 9)(x + 8)} - \frac{1(2x + 9)}{(x + 8)(2x + 9)}$$

Step 2: Subtract the numerators of the algebraic fractions. Simplify by expanding and collecting like terms.
Keep the denominator the same.

$$\frac{(5x - 3)(x + 8) - 1(2x + 9)}{(2x + 9)(x + 8)}$$

$$= \frac{(5x - 3)(x + 8) - 1(2x + 9)}{(2x + 9)(x + 8)}$$

$$= \frac{5x^2 + 40x - 3x - 24 - 2x - 9}{(2x + 9)(x + 8)}$$

$$= \frac{5x^2 + 35x - 33}{(2x + 9)(x + 8)}$$

Step 3: Try to factor the trinomial in the numerator. In this case, the trinomial is not factorable.

Final answer is $\dfrac{5x^2 + 35x - 33}{(2x + 9)(x + 8)}$

Example 3:
Simplify.

$$\frac{x^2 - 4x - 21}{x^2 - 6x - 7} \div \frac{x + 5}{x + 1}$$

Step 1: Factor the trinomials in the numerator and denominator of the first algebraic fraction.

$$x^2 - 4x - 21 = (x - 7)(x + 3)$$

$$x^2 - 6x - 7 = (x - 7)(x + 1)$$

Rewrite the trinomials with their factors.

$$\frac{(x - 7)(x + 3)}{(x - 7)(x + 1)} \div \frac{x + 5}{x + 1}$$

Step 2: Multiply the first algebraic fraction by the **reciprocal** of the second algebraic fraction.

$$\frac{(x - 7)(x + 3)}{(x - 7)(x + 1)} \cdot \frac{x + 1}{x + 5}$$

Step 3: Multiply the numerator by the numerator. Multiply the denominator by the denominator.

$$\frac{(x - 7)(x + 3)(x + 1)}{(x - 7)(x + 1)(x + 5)}$$

Step 4: Cancel out any factors common to the numerator and the denominator. These factors cancel out because anything divided by itself equals 1.

$$\frac{\cancel{(x - 7)}(x + 3)\cancel{(x + 1)}}{\cancel{(x - 7)}\cancel{(x + 1)}(x + 5)}$$

Final answer is $\frac{x + 3}{x + 5}$

Practice Questions:

30. Simplify.

$$\frac{2x^2 + x - 6}{2x^2 - x - 3}$$

a) $\frac{2x - 3}{x+1}$

b) $\frac{x - 6}{-x - 3}$

c) $\frac{2}{1}$

d) $\frac{x+2}{x+1}$

31. Subtract.

$$\frac{x-1}{x^2-3x+2} - \frac{3x}{x^2-1}$$

a) $\dfrac{-2x^2+6x-1}{x^3-2x^2-x+2}$

b) $\dfrac{6x-1}{x^3-x+2}$

c) $\dfrac{-2x-1}{-3x+3}$

d) $\dfrac{-3x+1}{x^3-2x^2-x+2}$

32. Simplify.

$$\frac{4x^2+2x}{x(2x-1)} \cdot \frac{3x-1}{2x+1}$$

a) $3x+2$

b) $\dfrac{3x-2}{x-1}$

c) $\dfrac{6x-2}{2x-1}$

d) $\dfrac{12x^2+2x-2}{4x^2-1}$

33. Simplify.

$$\frac{x}{x^2-x} \div \frac{10x}{x^2+x-2}$$

a) $\dfrac{10x^2}{x^2+2x}$

b) $\dfrac{10}{1+2x}$

c) $\dfrac{x+2}{10x}$

d) $\dfrac{10x}{x+2}$

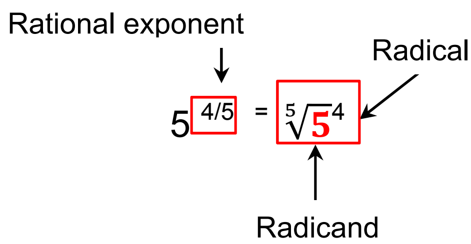
7. Radicals and positive rational exponents

Rational exponents are exponents that are fractions.

Radicals are expressions that have a square root, $\sqrt{}$, cube root, $\sqrt[3]{}$ or any n^th root, $\sqrt[n]{}$.

Radicand is the number or variable(s) that is/are beneath the radical sign.

Example:



Powers with positive rational exponents can be expressed as radicals according to the rule below.

$$x^{m/n} = \sqrt[n]{x^m} = (\sqrt[n]{x})^m$$

Notice that the denominator, **n**, of the rational exponent always goes on the outside of the root sign. The numerator, **m**, can either go with the radicand, x, OR it can be placed outside the brackets.

> Note: When there is no number written on the outside of the root sign, as in $\sqrt{}$, the number is assumed to be a 2 for the "square root", $\sqrt[2]{}$.

Example 1:

Write $4^{1/2}$ as a radical and then evaluate.

$4^{1/2} = \sqrt[2]{4^1}$

$= \sqrt[2]{4}$

$= 2$

Example 2:

Write $8^{1/3}$ as a radical and then evaluate.

$\mathbf{8^{1/3}} = \sqrt[3]{8^1}$

= $\sqrt[3]{8}$

= 2

Powers with fractional exponents can be evaluated using a scientific calculator.

Most calculators will have a y^x button for evaluating powers and ab/c button for entering fractions.

Radicals that are n^th roots can also be evaluated on a scientific calculator.

Most calculators will have a $\sqrt[x]{\ }$ button. (It is usually a second function button.)

Practice questions:

34. Evaluate $3^{2/5}$ using a calculator. Round to 3 decimal places.

35. Evaluate $(\sqrt[7]{8})^4$ using a calculator. Round to 3 decimal places.

Simplifying algebraic expressions with radicals

Algebraic or numerical expressions containing radicals can be simplified according to the rules below.

Product rule	$\sqrt[n]{x}\ (\sqrt[n]{y}) = \sqrt[n]{xy}$ where $x \geq 0$ and $y \geq 0$
Quotient rule	$\frac{\sqrt[n]{x}}{\sqrt[n]{y}} = \sqrt[n]{\frac{x}{y}}$ where $x \geq 0$ and $y \geq 0$
Addition rule	$a\sqrt[n]{x} + b\sqrt[n]{x} = (a+b)\sqrt[n]{x}$

Note: $\sqrt[n]{x} + \sqrt[n]{y}$ does NOT equal $\sqrt[n]{x+y}$

Tip: When working with numerical expressions involving radicals containing $\sqrt{\ }$, look for factors that are **perfect squares** that can be evaluated to give whole numbers.

Example:

Simplify.

$\mathbf{-5\sqrt{48} + 21\sqrt{3}}$

Step 1: Look for a perfect square that is a factor of 48.

48 = 16 x 3 and 16 is a perfect square.

Step 2: Write 48 as a product of 16 and 3. Apply the product rule.

$-5\sqrt{48} = -5\sqrt{16(3)} = -5\sqrt{16}\,(\sqrt{3})$

Step 3: Evaluate the perfect square.

$-5\sqrt{16}\,(\sqrt{3}) = -5(4)(\sqrt{3})$

Step 4: Simplify by multiplying −5 and 4.

$-5(4)(\sqrt{3}) = -20\sqrt{3}$

Step 5: Add the radicals since they have the same radicand.

$-20\sqrt{3} + 21\sqrt{3} = (-20 + 21)\sqrt{3}$

$= 1\sqrt{3}$

$= \sqrt{3}$

Practice Questions:

36. Simplify.

$\mathbf{-5\sqrt{8} + 3\sqrt{32}}$

a) $2\sqrt{2}$
b) $22\sqrt{8}$
c) $-15\sqrt{256}$
d) $-2\sqrt{40}$

37. Simplify.

$$\frac{\sqrt{81} \cdot \sqrt{63}}{3\sqrt{7}}$$

a) 27
b) 9

c) $\frac{1}{3\sqrt{7}}$

d) $\frac{1}{21\sqrt{7}}$

Example 2:

Evaluate.

$$\frac{\sqrt[3]{8}}{\sqrt{3}\cdot\sqrt{12}}$$

<u>Step 1:</u> Evaluate $\sqrt[3]{8}$ in the numerator since the answer is a whole number.

$\sqrt[3]{8} = 2$

<u>Step 2:</u> Use the product rule to simplify the denominator.

$\sqrt{3}\cdot\sqrt{12} = \sqrt{36}$

<u>Step 3:</u> Evaluate $\sqrt{36}$ since the answer is a whole number.

$\sqrt{36} = 6$

<u>Step 4:</u> Reduce the resulting fraction.

$\frac{\sqrt[3]{8}}{\sqrt{3}\cdot\sqrt{12}} = \frac{2}{6}$

$\frac{2}{6} = \frac{1}{3}$

Final answer is $\frac{1}{3}$.

Practice Question:

38. Evaluate.

$$\frac{\sqrt{8}}{\sqrt{2}}$$

a) $\frac{1}{2}$

b) 2

c) $\frac{2}{\sqrt{2}}$

d) $\frac{1}{\sqrt{2}}$

Appendix A: Glossary

Algebraic expression – a statement containing numbers, variables and mathematical operation signs

Binomial – an algebraic expression with two terms related to each other by a mathematical operation; (for example, $7x^3 + 9$)

Coefficient – the number multiplied by a variable or variables in an algebraic term; (for example, 3 is the coefficient in $3x^2y$)

Constant term – a term in a simplified algebraic expression that contains no variables and thus never changes; (for example, 4 is the constant term in the algebraic expression, $5x^3 - 18x + 4$)

Degree (of a term) – the sum of exponents on all the variables in a term

Difference – the result of subtraction

Difference of squares – a binomial in the form $ax^p - c$ where a and c are perfect squares and p is an even exponent

Equation – an equation uses an equal sign to state that two expressions are the same or equal to each other; (for example, $5x^2 + 18 = 25$)

Evaluate – to calculate the numerical value

Even – numbers that are divisible by 2 are considered to be even; these are 2, 4, 6, 8 and numbers that end in 0, 2, 4, 6, or 8

Expand – to eliminate brackets in an algebraic expression using the correct method of expanding

Exponent – the number in a power indicating how many times repeated multiplication is done

Factor – a number and/or variable that will divide into another number and/or variable exactly (for example, factors of $6x^2$ are x, x^2, 1, 2, 3, and 6)

To **factor** (an algebraic expression) − to find the numbers, monomials, binomials or polynomials that multiplied together result in the given algebraic expression

Fraction – a rational number representing part of a whole

Greatest common factor – the greatest factor (consisting of numbers and/or variables) that ALL the terms in a given algebraic expression are divisible by; (for example, the greatest common factor of $27x^3y$ and $36xy^2$ is 9xy)

Like terms – terms with the same variables AND exponents on those variables; the coefficients of like terms may be different; (for example, $4y^3x$ and $-9xy^3$ are like terms, but not $10y^2x$)

Linear equation – an equation with degree 1 (i.e. 1 is the highest exponent on any one variable);

(for example, $3x + 2 = 10$)

Monomial – an algebraic expression with one term; (for example, $7x^3$)

Perfect square – a number whose square root is a whole number and/or a variable with an even exponent

Perfect square trinomial – a trinomial in the form $ax^2 + bx + c$ where a and c are perfect squares and $b = 2\sqrt{a}\sqrt{c}$; (for example, $9x^2 + 30x + 25$)

Polynomial – an algebraic expression with many terms related to each other by mathematical operations; (for example, $7x^3 + 5x^2 + 18xy - 9$)

Power – a number raised to an exponent

Product – the result of multiplication

Radical – an expression that has a square root, $\sqrt{\ }$, cube root, $\sqrt[3]{\ }$ or any n^{th} root, $\sqrt[n]{\ }$

Radicand – the number or variable(s) that is/are beneath the radical sign; (for example, 5 is the radicand in $\sqrt[3]{5}$)

Rational – written in the form $\frac{a}{b}$ where *a* is not a multiple of *b* and *b* does not equal 0

Reciprocal – the multiplicative inverse of a number; the product of two reciprocals is by definition equal to 1. For a fraction $\frac{a}{b}$, the reciprocal is $\frac{b}{a}$.

Simplify – to write an algebraic expression in simplest form; an expression is in simplest form when there are no more like terms that can be combined

Solve – to find the numerical value

Squared – raised to the exponent 2

Square root – the square root of a number is the value of the number, which multiplied by itself gives the original number

Sum – the result of addition

Trinomial – an algebraic expression with three terms related to each other by mathematical operations; (for example, $7x^3 + 5x - 9$)

Variable – a letter of the alphabet used to represent an unknown number or quantity

Variable term – a term in a simplified algebraic expression that contains variables; (for example, $5x^3$ and $-18x$ are variable terms in the algebraic expression, $5x^3 - 18x + 4$)

Appendix B: Answers to Practice Questions

Topic	Page Number	Question Number	Answer
Evaluating algebraic expressions	4	**1.**	b
	4	**2.**	c
Solving linear equations	7	**3.**	a
	7	**4.**	c
Addition and subtraction of polynomials	8	**5.**	b
	9	**6.**	d
	9	**7.**	b
	9	**8.**	c
Multiplication of polynomials	11	**9.**	d
	11	**10.**	b
	12	**11.**	a
	13	**12.**	d
	14	**13.**	d
	14	**14.**	a
	16	**15.**	c
	16	**16.**	c
Factoring: - Common factoring - Factoring $ax^2 + bx + c$ when $a = 1$ - Factoring $ax^2 + bx + c$ when $a \neq 1$ - Difference of squares - Perfect square trinomial	18	**17.**	a
	18	**18.**	d
	20	**19.**	b
	20	**20.**	c
	23	**21.**	c
	23	**22.**	a
	24	**23.**	d
	26	**24.**	d
	26	**25.**	c
	27	**26.**	a
	29	**27.**	b
	29	**28.**	d
	29	**29.**	c
Operations with algebraic fractions	32	**30.**	d
	33	**31.**	a
	33	**32.**	c
	33	**33.**	c
Radicals and positive rational exponents	35	**34.**	1.552
	35	**35.**	3.281

Radicals and positive rational exponents *(…continued)*	36	**36.**	a
	36	**37.**	b
	37	**38.**	b