ACCUPLACER Arithmetic Assessment Preparation Guide

Please note that the guide is for reference only and that it does not represent an exact match with the assessment content. The Assessment Centre at George Brown College is not responsible for students’ assessment results.

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Introduction

The Accuplacer Arithmetic Assessment Preparation Guide is a reference tool for George Brown College students preparing to take the Accuplacer Arithmetic assessment. The study guide focuses on foundation-level math skills. The study guide does not cover all topics on the assessment.

The Accuplacer Arithmetic Assessment Preparation Guide is organized around a select number of topics in arithmetics. It is recommended that users follow the guide in a step-by-step order as one topic builds on the previous one. Each section contains theory, examples with full solutions and practice question(s). Answers to practice questions can be found in Appendix D.

Reading comprehension and understanding of terminology are an important part of learning mathematics. Appendix A has a glossary of mathematical terms used throughout the Study Guide.
1. Operations with Whole Numbers

Addition of Whole Numbers (by hand):

To add two whole numbers by hand, first line up the numbers according to place values (See Glossary in Appendix A for definition of place values). Perform the addition one column at a time starting at the right. When the sum of the digits in any column is greater than 9, the digit in the tens place value must be carried over to the next column.

Example:

Evaluate. 5789 + 232.

```
   thousands  tens
     5 7 8 9
   + 2 3 2
  _________
     6 0 2 1
```

Final answer is 6021.

Practice Questions:

1. Calculate 987 + 1435.

2. Calculate 15369 + 10899.

Note: The same procedure may be used with more than two numbers.
Subtraction of Whole Numbers (by hand):

To subtract two whole numbers by hand, first line up the numbers according to place values. Perform the subtraction one column at a time, starting at the right. If the number on the top is smaller than the number on the bottom, you can regroup 1 from the number in the next column to the left and add 10 to your smaller number.

Example:

Evaluate. 6012 – 189.

<table>
<thead>
<tr>
<th>thousands</th>
<th>tens</th>
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<tbody>
<tr>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
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<table>
<thead>
<tr>
<th>hundreds</th>
<th>ones</th>
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<tbody>
<tr>
<td>1</td>
<td>9</td>
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<tr>
<td>8</td>
<td>3</td>
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Step 1: 2 – 9 would give us a negative number. Thus, we need to regroup 1 from the tens place value; add 10 to 2 to get 12. 12 – 9 = 3
Write 3 in the ones place value.
Note: In the tens place value, we have 0 left since we took 1 from 1.

Step 2: 0 – 8 would again give us a negative number. Thus, we need to regroup 1 from the hundreds place value, which is the next column to the left. The hundreds place value contains a 0, so we go to the next column to the left and regroup 1 from the thousands place value. This gives 10 in the hundreds place value and now we can borrow 1 from the hundreds, leaving 9 left over. Add 10 to 0 in the tens place values to get 10.
10 – 8 = 2
Write 2 in the tens place value.
In the hundreds place value we have 9 left over and we have 5 left over in the thousands place value.

Step 3: 9 – 1 = 8
Write 8 in the hundreds place value.

Step 4: Write 5 in the thousands place value.

Final answer is 5823.

Practice Questions:


Multiplication of Whole Numbers (by hand):

To multiply two whole numbers by hand, first line up the numbers according to the last column. Next, multiply the number in the bottom row by each digit in the number in the top row, starting with the right-most digit (Note: you MUST know the multiplication table for this step. A multiplication table can be found in Appendix B). If the multiplication results in a two-digit number (i.e. a number greater than 9), regroup the tens place value with the next column by adding it to the product. Furthermore, every time you start a new row in your answer, starting with the second row, you need to indent one place value. Lastly, add all of the separate products to get the final answer.

Example:

Evaluate. 5327 x 129

Step 1: Line up the numbers according to the correct place values (ones, tens, hundreds, etc.)

Step 2: 9 x 7 = 63
Write 3 in the ones place value. Regroup 6 with the next place value.

Step 3: 9 x 2 = 18
18 + 6 = 24
Write 4 in the hundreds place value. Regroup 2 with the next place value.

Step 4: 9 x 3 = 27
27 + 2 = 29
Write 9 in the thousands place value. Regroup 2 with the next place value.

Step 5: 9 x 5 = 45
45 + 2 = 47
Write 7 in the thousands place value. Write 4 in the ten thousands place value.

Step 6: Indent one place value on the second row of multiplication. We indent because we are now multiplying 5327 by the 2 in the tens place value. (Instead of indenting you may also write a 0 in the ones place value of the second row.)

Step 7: 2 x 7 = 14
Write 4 in the tens place value. Regroup 1 to the next place value.

Step 8: 2 x 2 = 4
4 + 1 = 5
Write 5 in the hundreds place value.

Step 9: 2 x 3 = 6
Write 6 in the thousands place value.

Step 10: 2 x 5 = 10
Write 0 in the ten thousands place value and 1 in the hundred thousands place value.

Final Answer: 106543
Final answer is 687183.

Practice Questions:

5. Evaluate. 2369 x 152.

6. Evaluate. 1078 x 691.

Division of Whole Numbers (by hand):

To divide two whole numbers by hand, set up the dividend and divisor as shown below. The quotient (the answer) will go on the line on top of the dividend.

\[
\text{quotient} \quad \underline{\text{divisor}} \quad \underline{\text{dividend}}
\]

Example:

Evaluate. 456 ÷ 3.

Step 1: Set up the divisor and dividend as shown above. 456 is the dividend; 3 is the divisor.

Step 2: How many times does 3 go into 4 evenly? Once. Therefore, put 1 in the quotient. 3 x 1 = 3 Subtract 3 from 4. 1 remains.
Step 3: Bring the next number down, in this case 5. How many times does 3 go into 5 evenly? Five times. Therefore, put 5 in the quotient. 
   \[ 3 \times 5 = 15 \]
   Subtract 15 from 15. Remainder is 0.

Step 4: Bring the next number, 6, down. How many times does 3 go into 6 evenly? Twice. Therefore, put 2 in the quotient. 
   \[ 3 \times 2 = 6 \]
   Subtract 6 from 6. Remainder is 0 and there are no more numbers to divide 3 into. Thus, the division is done.

Final answer is 152.

Note: Division problems with two-digit divisors follow the same method as shown above.
Note: For division problems where the dividend does not divide evenly by the divisor, the quotient will have a remainder.

Practice Questions:

7. Calculate 12192 ÷ 8.

8. Calculate 115620 ÷ 12.

Order of Operations

As a general rule of thumb, the acronym BEDMAS can be followed for the correct order of operations.

- **Brackets**
- **Exponents** → Do in order from left to right.
- **Division** → Do in order from left to right.
- **Multiplication** → Do in order from left to right.
- **Addition** → Do in order from left to right.
- **Subtraction** → Do in order from left to right.
Important notes:

- If there are **multiple exponents**, evaluate the powers from left to right as they appear in the question.
- If there are **multiple brackets**, it does not matter which ones you do first, second, etc.
- If there are **multiple operations within brackets**, do the operations according to BEDMAS.
- **Division and multiplication** is done from left to right. This means that multiplication should be done before division, if it appears to the left of division.
- **Addition and subtraction** is done from left to right. Subtraction should be done first, if it is to the left of addition.
- For **rational expressions**, the numerator and denominator are evaluated separately according to BEDMAS. Then, determine the quotient.

Examples:

Evaluate.

a) \(9 + \frac{(10 - 8)^2}{2 \times 3}\) \hspace{1cm} \text{Step 1: Do the operation inside the brackets.}

\[= 9 + \frac{2^2}{2 \times 3}\]  \hspace{1cm} \text{Step 2: Evaluate the exponent.}

\[= 9 + \frac{4}{2 \times 3}\]  \hspace{1cm} \text{Step 3: Do the division since it appears first going from left to right.}

\[= 9 + \frac{2 \times 3}{3}\]  \hspace{1cm} \text{Step 4: Do the multiplication.}

\[= 9 + 6\]  \hspace{1cm} \text{Step 5: Do the addition.}

\[= 15\]  \hspace{1cm} \text{Final answer is 15.}

b) \(\frac{2^3 + 12 + 4}{9(3) - 12}\) \hspace{1cm} \text{Step 1: Evaluate the exponent on the top. Do the multiplication on the bottom.}

\[= \frac{8 + 12 + 4}{27 - 12}\]  \hspace{1cm} \text{Step 2: Do the division on the top.}

\[= \frac{8 + 3}{27 - 12}\]  \hspace{1cm} \text{Step 3: Do the addition on the top. Do the subtraction on the bottom.}

\[= \frac{11}{15}\]  \hspace{1cm} \text{This is the final answer. The fraction cannot be reduced.}
Practice Question:

9. Evaluate. $3 + 8 \times (9 - 2)^2$
   
   a) 115  
   b) 248  
   c) 56  
   d) 395

2. Operations with Fractions

A fraction is a number that represents parts of a whole. The numerator represents the number of parts you have. The denominator is the number of parts into which one whole is divided.

Example:

\[
\text{Numerator} \quad \longrightarrow \quad 5 \quad \longleftrightarrow \quad \text{You have 5 parts} \\
\text{Denominator} \quad \longrightarrow \quad \frac{7}{7} \quad \longleftrightarrow \quad \text{One whole is divided into 7 parts}
\]

An improper fraction is a fraction whose numerator is bigger than the denominator (e.g. $\frac{5}{3}$, $\frac{9}{2}$).

This means that an improper fraction is greater than one whole.

A mixed number consists of a whole number and a fraction. (e.g. $2 \frac{7}{8}$).

A mixed number can be written as an improper fraction and vice versa.

Converting a Mixed Number to an Improper Fraction

To convert a mixed number to an improper fraction:

- Multiply the whole number by the denominator and add to the numerator*.
- Keep the denominator the same**.

*This tells us how many parts we have in total (the parts that make up the whole number(s) and those in the numerator).

**We keep the denominator the same because we have NOT changed the number of parts one whole is divided into.

Example:
Convert $2\frac{7}{8}$ to an improper fraction.

Multiply whole number (2) by the denominator (8). Add the numerator (7).

$2\frac{7}{8} = \frac{2(8)+7}{8} = \frac{23}{8}$

Keep the denominator (8) the same

Practice Question:

10. Which of the following fractions is equivalent to $5\frac{7}{9}$?

   a) $\frac{35}{45}$
   b) $\frac{12}{9}$
   c) $\frac{52}{45}$
   d) $\frac{52}{9}$

Converting an Improper Fraction to a Mixed Number

To convert an improper fraction to a mixed number:
- Divide the numerator by the denominator. The answer should be a whole number and a remainder.
- The whole number is the whole number part of the mixed number.
- The remainder is the numerator of the fractional part of the mixed number.
- Keep the denominator the same.

Example:

Convert $\frac{9}{2}$ into a mixed number.

2 goes into 9 four times evenly. Thus, our mixed number will have 4 wholes and whatever fraction part is left over.

$\frac{9}{2} = 4\frac{1}{2}$

Thus, we have $\frac{1}{2}$ left over as the fractional part of the mixed number.
Practice question:

11. What is $\frac{36}{7}$ written as a mixed number?

a) $\frac{1}{7}$

b) $5\frac{1}{7}$

c) $6\frac{1}{7}$

d) $5\frac{1}{5}$

Equivalent Fractions

Two fractions are considered equivalent if they are equal in value.

To determine if two given fractions are equivalent, find the lowest terms of each fraction. If the lowest terms are the same, then the fractions are equivalent. To find the lowest terms of a fraction, divide both the numerator and denominator by their greatest common factor.

Example:

$\frac{1}{2}$ is equivalent to $\frac{2}{4}$.

In the diagram below, you can see that $\frac{1}{2}$ and $\frac{2}{4}$ are equal in value. The large rectangle represents one whole. The first rectangle is divided into two equal parts (denominator = 2) and one part is coloured (numerator = 1). The second rectangle is divided into four equal parts (denominator = 4) and two parts are coloured (numerator = 2). The coloured parts occupy the same area in both diagrams because the fractions are equal in value.

However, $\frac{1}{2}$ is NOT equivalent to $\frac{2}{3}$.
As you can see in the diagram above, \( \frac{2}{3} \) occupies more space than \( \frac{1}{2} \). This is because \( \frac{2}{3} \) is bigger than \( \frac{1}{2} \) and the two fractions are NOT equivalent (i.e. not equal in value).

Also, notice that \( \frac{2}{4} \) can be reduced to \( \frac{1}{2} \) by dividing both the numerator and denominator of \( \frac{2}{4} \) by 2. On the contrary, \( \frac{2}{3} \) can NOT be reduced to \( \frac{1}{2} \).

\[
\frac{2 \div 2}{4 \div 2} = \frac{1}{2}
\]

Therefore, \( \frac{2}{4} = \frac{1}{2} \)

But \( \frac{1}{2} \neq \frac{2}{3} \)

**Note:**

For any fraction, there is an infinite number of equivalent fractions.

Given a fraction, an equivalent fraction can be obtained by multiplying both the numerator and denominator by the same number.

**Example:**

**Write two fractions that are equivalent to \( \frac{2}{5} \).**

\[
\frac{2 \cdot 3}{5 \cdot 3} = \frac{6}{15}
\]

\[
\frac{2 \cdot 4}{5 \cdot 4} = \frac{8}{20}
\]

Therefore, \( \frac{6}{15} \) and \( \frac{8}{20} \) are equivalent to \( \frac{2}{5} \).

Note that both \( \frac{6}{15} \) and \( \frac{8}{20} \) can be reduced to \( \frac{2}{5} \).

**Practice question:**

**12. Which of the following fractions are equivalent to \( \frac{7}{9} \)?**

a) \( \frac{14}{21} \)

b) \( \frac{28}{35} \)

c) \( \frac{21}{27} \)

d) none of the above
Addition of Fractions

- To add two fractions, they MUST have the same (common) denominator. Usually, it is best to find the lowest common denominator.
- The lowest common denominator is equal to the lowest common multiple of the two denominators. If one of the denominators is a multiple of another, than the lowest common multiple is the bigger denominator.
- Once you have a common denominator, change each fraction to an equivalent fraction with the desired denominator.
- Lastly, add the numerators of the fractions and keep the denominator the same.
- Reduce the fraction to lowest terms and/or change an improper fraction to a mixed number, if necessary.

Examples:

1) What is the sum of $\frac{2}{3}$ and $\frac{5}{6}$?

\[
\frac{2}{3} + \frac{5}{6} = \]

Step 1: Since the denominators of the fractions are different, we first need to find the lowest common denominator. In this case, the lowest common denominator is 6, since 6 is a multiple of 3.

\[
\frac{2}{3} = \frac{2 \cdot 2}{3 \cdot 2} = \frac{4}{6}
\]

Step 2: $\frac{2}{3}$ needs to be changed into an equivalent fraction with a denominator of 6. To do this, multiply the numerator, 2, and denominator, 3, by 2. The fraction, $\frac{5}{6}$, can stay as is, since it already has a denominator of 6.

\[
\frac{4}{6} + \frac{5}{6} = \frac{4+5}{6} = \frac{9}{6}
\]

Step 3: Add the numerators. Keep the denominator the same.

\[
\frac{9}{6} = \frac{9 \div 3}{6 \div 3} = \frac{3}{2}
\]

Step 4: $\frac{9}{6}$ can be reduced to $\frac{3}{2}$ by dividing both the numerator and denominator by 3.

\[
\frac{3}{2} = 1\frac{1}{2}
\]

Step 5: $\frac{3}{2}$ is an improper fraction and should be changed into a mixed number.
2) \[
\frac{3}{5} + \frac{7}{9} = ?
\]

First find the lowest common denominator. We are looking for the lowest common multiple of 5 and 9.

*Note: If you are having trouble finding the lowest common multiple of two or more numbers, list the multiples for each number in a list. Look for the lowest multiple that appears in your lists.*

**Multiples of 5:**

5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 65, …

**Multiples of 9:**

9, 18, 27, 36, 45, 54, 63, 72, …

The lowest common multiple of 5 and 9 is 45.

In order to make \(\frac{3}{5}\) into an equivalent fraction with a denominator of 45, multiply both the numerator and the denominator by 9.

\[
\frac{3}{5} = \frac{3 \times 9}{5 \times 9} = \frac{27}{45}
\]

In order to make \(\frac{7}{9}\) into an equivalent fraction with a denominator of 45, multiply both the numerator and the denominator by 5.

\[
\frac{7}{9} = \frac{7 \times 5}{9 \times 5} = \frac{35}{45}
\]

\[
\frac{27}{45} + \frac{35}{45} = \frac{62}{45}
\]

\[
\frac{62}{45} = 1\frac{17}{45}
\]

3) \[
\frac{3}{16} + \frac{7}{12} = ?
\]

Find the lowest common denominator. We are looking for the lowest common multiple of 16 and 12.

**Multiples of 16:**

16, 32, 48, 64, 80, 96, …

**Multiples of 12:**

12, 24, 36, 48, 60, 72, …

The lowest common multiple of 16 and 12 is 48.

In order to make \(\frac{3}{16}\) into an equivalent fraction with a denominator of 48, multiply both the numerator and the denominator by 3.

\[
\frac{3}{16} = \frac{3 \times 3}{16 \times 3} = \frac{9}{48}
\]
In order to make \( \frac{7}{12} \) into an equivalent fraction with a denominator of 48, multiply both the numerator and the denominator by 4. \( \frac{7}{12} = \frac{7 \times 4}{12 \times 4} = \frac{28}{48} \)

\[ \frac{9}{48} + \frac{28}{48} = \frac{37}{48} \]

\( \frac{37}{48} \) cannot be reduced.

Therefore, the sum is \( \frac{37}{48} \).

Practice questions:

13. What is the sum of \( \frac{2}{5} \) and \( \frac{1}{15} \)?
   a) \( \frac{3}{20} \)
   b) \( \frac{7}{15} \)
   c) \( \frac{3}{15} \)
   d) \( \frac{7}{20} \)

14. What is the sum of \( \frac{1}{7} \) and \( \frac{3}{5} \)?
   a) \( \frac{11}{35} \)
   b) 1
   c) \( \frac{26}{35} \)
   d) \( \frac{4}{12} \)

Subtraction of Fractions

To subtract two fractions, they **MUST have the same (common) denominator** (this is the same as adding fractions).

Once you have a common denominator, change each fraction to an equivalent fraction with the desired denominator.

Lastly, **subtract the numerators of the fractions** and keep the denominator the same. In subtraction, the order matters! Keep it the order the same in how the question is written.

**Reduce** the fraction to lowest terms and/or change an improper fraction to a mixed number if necessary.

Example:
What is the difference between $\frac{13}{20}$ and $\frac{7}{50}$?

First, we need to find a common denominator for the two fractions. The lowest common multiple of 20 and 50 is 100.

\[
\frac{13}{20} = \frac{13 \times 5}{20 \times 5} = \frac{65}{100}
\]

\[
\frac{7}{50} = \frac{7 \times 2}{50 \times 2} = \frac{14}{100}
\]

Now, subtract the numerators and keep the denominator the same.

\[
\frac{65}{100} - \frac{14}{100} = \frac{51}{100}
\]

$\frac{51}{100}$ cannot be reduced.

The final answer is $\frac{51}{100}$.

Practice question:

15. What is the difference between $\frac{23}{63}$ and $\frac{2}{9}$?
   a) $\frac{21}{54}$
   b) $\frac{1}{7}$
   c) $\frac{21}{63}$
   d) $\frac{8}{63}$

**Multiplication of Fractions**

To multiply fractions:
- Multiply the numerator by the numerator
- Multiply the denominator by the denominator
- Reduce and/or change to a mixed number, if you can.

Note: a common denominator is NOT needed!

If multiplying a mixed number by a fraction OR two mixed numbers, first change the mixed number(s) to improper fraction(s) and then follow the same steps as above.
Examples:

1) What is the product of \( \frac{13}{15} \) and \( \frac{7}{5} \)?

First, multiply the numerator by the numerator and denominator by the denominator.

\[
\frac{13}{15} \times \frac{7}{5} = \frac{13(7)}{15(5)} = \frac{91}{75}
\]

The product is an improper fraction. Change it to a mixed number.

\[
\frac{91}{75} = 1 \frac{16}{75}
\]

Final answer is \( 1 \frac{16}{75} \).

2) What is the product of \( \frac{19}{20} \times \frac{4}{9} \)?

First, multiply the numerator by the numerator and denominator by the denominator.

\[
\frac{19}{20} \times \frac{4}{9} = \frac{19(4)}{20(9)} = \frac{76}{180}
\]

Now, reduce the product. 76 and 180 are both divisible by 4.

\[
\frac{76}{180} \div 4 = \frac{19}{45}
\]

Note: It is also possible to reduce our fractions BEFORE we do the multiplication. This can be advantageous because the reducing happens with smaller numbers.

In our multiplication above, we can reduce 4 on the top and 20 on the bottom by dividing both numbers by 4. We can reduce a numerator from one fraction and denominator from another because order does not matter when multiplying numbers (i.e. 19 x 4 or 4 x 19 gives the same answer).

\[
\frac{19}{4} \times \frac{1}{\frac{9}{5}} = \frac{19}{45}
\]

3) What is the product of \( 2\frac{1}{9} \) and \( \frac{3}{5} \)?

Our first step is to change \( 2\frac{1}{9} \) into an improper fraction.
2\frac{1}{9} = \frac{19}{9}

Now, we can multiply numerator by the numerator and denominator by the denominator. We can also reduce before performing the actual multiplication (3 and 9 are both divisible by 3).

\[
\frac{19}{9} \times \frac{3}{5} = \frac{19(\div 3)}{9(\div 3)} = \frac{19}{3(\div 3)} = \frac{19}{15}
\]

Since our product is an improper fraction, we need to change it to a mixed number.

\[
\frac{19}{15} = 1\frac{4}{15}
\]

The final answer is \(1\frac{4}{15}\).

Practice questions:

16. What is the product of \(\frac{4}{19}\) and \(\frac{5}{2}\)?
   a) \(\frac{20}{36}\)
   b) \(\frac{20}{21}\)
   c) \(\frac{10}{19}\)
   d) \(\frac{8}{95}\)

17. What is the product of \(17\frac{1}{5}\) and \(2\frac{2}{3}\)?
   a) \(\frac{45}{15}\)
   b) \(\frac{40}{15}\)
   c) \(\frac{11}{30}\)
   d) \(\frac{34}{20}\)

Division of fractions

In order to divide two fractions:

- Change the division problem to a multiplication problem by multiplying by the reciprocal of the divisor. (To find the reciprocal of a fraction, switch the numerator and denominator).
- Multiply the fractions (see above).
If dividing a mixed number by a fraction OR two mixed numbers, first change the mixed number(s) to improper fraction(s) and then follow the same steps as above.

**Examples:**

1) What is the quotient of $\frac{19}{20}$ and $\frac{1}{5}$?

In order to divide $\frac{19}{20}$ and $\frac{1}{5}$, we need to multiply $\frac{19}{20}$ by the reciprocal of $\frac{1}{5}$.

The reciprocal of $\frac{1}{5}$ is $\frac{5}{1}$.

$$\frac{19}{20} \div \frac{1}{5} = \frac{19}{20} \times \frac{5}{1}$$

$$\frac{19}{20} \times \frac{5*1}{1} = \frac{19(1)}{4(1)} = \frac{19}{4}$$

The quotient is an improper fraction so we need to change it to a mixed number.

$$\frac{19}{4} = 4\frac{3}{4}$$

Final answer is $4\frac{3}{4}$.

2) What is the quotient of $3\frac{6}{7}$ and $\frac{9}{11}$?

First, we need to change $3\frac{6}{7}$ into an improper fraction.

$$3\frac{6}{7} = \frac{27}{7}$$

Now, we divide $\frac{27}{7}$ and $\frac{9}{11}$ by multiplying $\frac{27}{7}$ by the reciprocal of $\frac{9}{11}$.

$$\frac{27}{7} \div \frac{9}{11} = \frac{27}{7} \times \frac{11}{9}$$

$$\frac{27}{7} \times \frac{11*3}{9} = \frac{3(11)}{7(1)} = \frac{33}{7}$$

Last step is to change $\frac{33}{7}$ to a mixed number.
Practice questions:

18. What is the quotient of $\frac{5}{11}$ and $\frac{4}{33}$?
   a) $\frac{15}{4}$
   b) $\frac{20}{363}$
   c) $\frac{3}{4}$
   d) Both a and c

19. What is the quotient of $\frac{73}{90}$ and $3\frac{3}{5}$?
   a) $\frac{73}{324}$
   b) $\frac{219}{150}$
   c) $\frac{1314}{150}$
   d) $\frac{324}{73}$

Ordering Fractions

There are a numbers of ways to compare (greater than, less than, equal to) or order fractions.

One way is to convert all fractions to equivalent fractions with a common denominator and then to compare the numerators. Given the same denominator, a fraction with a bigger numerator will be greater than a fraction with a smaller numerator.

Another approach is to convert all fractions to decimals and then use the decimals to compare or order the given fractions. (See section on converting fractions to decimals.)

Example:

Order the following numbers from least to greatest.

$\frac{2}{3}, \frac{5}{6}, \frac{7}{12}, 2, 1\frac{1}{2}$

Method 1: Using a common denominator
The lowest common multiple of 3, 6, 12, 1 and 2 is 12. Thus, the lowest common denominator for the fractions is 12.

\[
\frac{2}{3} = \frac{8}{12} \\
\frac{5}{6} = \frac{10}{12} \\
2 = \frac{2}{1} = \frac{24}{12} \\
1 \frac{1}{2} = \frac{3}{2} = \frac{18}{12} \\
\frac{7}{12} < \frac{8}{12} < \frac{10}{12} < \frac{18}{12} < \frac{24}{12}
\]

Therefore,

\[
\frac{7}{12} < \frac{2}{3} < \frac{5}{6} < 1 \frac{1}{2} < 2
\]

**Method 2: Converting fractions to decimals**

\[
\frac{2}{3} = 0.666666667 \\
\frac{5}{6} = 0.833333333 \\
\frac{7}{12} = 0.583333333 \\
1 \frac{1}{2} = 1.5
\]

\[
0.583333333 < 0.666666667 < 0.833333333 < 1.5 < 2
\]

Therefore,

\[
\frac{7}{12} < \frac{2}{3} < \frac{5}{6} < 1 \frac{1}{2} < 2
\]

**Practice Question:**

20. Which of the following numbers are greater than \(\frac{7}{15}\)?

i) \(\frac{1}{2}\)  ii) \(\frac{1}{3}\)  iii) \(\frac{4}{5}\)  iv) \(\frac{9}{10}\)

a) i and ii 

b) i, ii and iii 

c) ii and iv 

d) i, iii and iv
3. Operations with Decimals

Addition (by hand)

To add two decimals by hand, first line up the decimals according to place values. The decimal points should be lined up one on top of another. Perform the addition one column at a time starting at the right. The number in the tens place value of any two-digit numbers must be regrouped with the next column. The decimal point in the answer goes directly underneath the decimal points in the numbers being added. This is why it is so important to line up the decimals correctly!

Example: 96.45 + 3.987

```
  96.45
+  3.987
```

```
  96
+  39
```

```
  45
+  87
```

```
  111
```

```
  100.437
```

Make sure to line up the place values according to the decimal point. The decimal point will go in the SAME position in your answer.

If you wish, you may write 0 to fill in the empty place value.

9 + 1 = 10
Write 0. Regroup 1 to the left.

6 + 3 + 1 = 10
Write 0. Regroup 1 to the left.

4 + 9 + 1 = 14
Write 4. Regroup 1 to the left.

5 + 8 = 13
Write 3. Regroup 1 to the left.

7 + 0 = 7
Practice questions:

21. Add 35.879 + 1.36

22. Add 0.0369 + 1.099

Subtraction (by hand)

To subtract two decimals by hand, first line up the decimals according to place values. The decimal points should be lined up one on top of another. Perform the subtraction one column at a time starting at the right. If the number on the top is smaller than the number on the bottom, you can regroup 1 from the number in the next column to the left and add 10 to your smaller number. The decimal point in the answer goes directly beneath the decimal points in the numbers being subtracted. Any “empty” decimal place values can be filled in with 0’s.

Example: 16.9 – 7.804

```
  1 6 . 9
- 7 . 8 0 4
```

```
  1 6 . 9 0 0
- 7 . 8 0 4
```

```
  9 . 0 9 6
```

Make sure to line up the place values according to the decimal point. The decimal point will go in the SAME position in your answer.
Practice questions:

23. Subtract 369.8 – 1.539

24. Subtract 10.003 – 1.678

Multiplication (by hand)

To multiply two decimals by hand, first line up the numbers according to the last column. Ignore the decimal points for now. Next, follow the same procedure as multiplying whole numbers. Lastly, count the total number of decimal places between the two decimals. Starting at the right most decimal place in the answer, count the same number of decimal places going left. Place the decimal point here. The number of decimal places in the answer should be the same as the total number of decimal places in the question.

Example:

Evaluate. 98.76 x 1.305.

```
  98.76
x 1.305
```

Step 1: Line up the numbers according to the last column. Ignore the decimal point and place values!

```
  98.76
x 1.305
  49380
```

Step 2: Multiply 5 by 9876. Line up the product with the numbers in the question.

```
  98.76
x 1.305
  49380
  0000
```

Step 3: Multiply 0 by 9876. Indent the product one space to the left.

```
  98.76
x 1.305
  49380
  0000
```

Step 4: Multiply 3 by 9876. Indent the product two spaces to the left.

```
  98.76
x 1.305
  49380
  0000
  29628
```
Step 5: Multiply 1 by 9876. Indent the product three spaces to the left.

Step 6: Add the products column by column starting at the right.

Step 7: Count the total number of decimal places in the numbers being multiplied, five in this example.

Your answer will have the same number of decimal places. Starting at the right-most digit, count the appropriate number of spaces and place the decimal point.

The final answer is 128.8818. The last 0 can be dropped as this does not change the value (or magnitude) of the number.

Practice questions:

25. Multiply 12.89 by 3.671

26. Multiply 0.0031 by 127.9
**Division (by hand)**

One way to divide two decimals is to make the divisor a whole number. To do this, count the number of decimal places in the divisor and multiply by a multiple of 10 that will make the divisor a whole number (Rule of thumb: the number of zeros in the multiple of 10 should equal the number of decimal places you are trying to get rid of). For example, 12.4 should be multiplied by 10 since multiplying by 10 moves the decimal place one place to the right. 12.4 x 10 = 124. However, in order to keep the division problem equivalent to the question given, you MUST also multiply the dividend by the same multiple of 10! Note that, the dividend can remain a decimal. For example, to divide 62.08 by 3.2, multiply the dividend and divisor by 10. The division problems becomes 620.8 ÷ 32. Once the divisor is a whole number, proceed to do the division problem as you would divide any two whole numbers. If the dividend is a decimal, put a decimal point in the quotient once you “meet” the decimal point in the dividend during long division (see Example 2).

**Example:**

Evaluate. 358.8 ÷ 1.2

1.2 x 10 = 12 \hspace{1cm} \textbf{Step 1:} You want to make the divisor, 1.2, into a whole number. Since 1.2 has one decimal place, multiply by 10.

358.8 x 10 = 3588 \hspace{1cm} \textbf{Step 2:} Since 1.2 was multiplied by 10, the dividend, 358.8 MUST also be multiplied by 10.

The division problem becomes 3588 ÷ 12.

\[
\begin{array}{c|c}
299 \\
12 & 3588 \\
-24 & \\
118 & \\
108 & \\
108 & \\
0 & \\
\end{array}
\]

\textbf{Step 3:} Do long division as you would normally divide two whole numbers.

Final answer is 299.

**Example 2:**

Evaluate. 21.6279 ÷ 0.07.

0.07 x 100 = 7 \hspace{1cm} \textbf{Step 1:} You want to make the divisor, 0.07, into a whole number. Since 0.07 has two decimal places, multiply by 100.
21.6279 \times 100 = 2162.79 \quad \text{Step 2: Since 0.07 was multiplied by 100, the dividend,} \\
\quad \text{21.6279 MUST also be multiplied by 100.}

The division problem becomes 2162.79 ÷ 7.

\[
\begin{array}{c|ccccc}
7 & 2162.79 \\
\hline
\quad & 308.27 \\
\quad & 21 \\
\quad & 56 \\
\quad & 67 \\
\quad & 49 \\
\quad & 0 \\
\end{array}
\]

\text{Step 3: Do long division as you would normally divide two whole numbers.}

\text{Once you “meet” the decimal point in the dividend, put a decimal point in the quotient.}

\text{Put a decimal point in the quotient before bringing the 7 down.}

Practice questions:

27. Evaluate. \(20.79 ÷ 1.1\)

28. Evaluate. \(0.08265 ÷ 0.05\)

5. Conversions between Fractions, Decimals and Percent

Converting Fractions to Decimals

\text{To convert a proper or improper fraction into a decimal, divide the numerator by the denominator using long division or a calculator.}

The decimal equivalent of a proper fraction will always be less than 1 but greater than 0. The decimal equivalent of an improper fraction will always be greater than 1.

\text{Example:}

\text{Convert } \frac{6}{15} \text{ into a decimal.}

\[6 ÷ 15 = 0.4\]

\text{To convert a mixed number into a decimal, divide the numerator by the denominator for the fractional part and then add the whole number to the resulting decimal.}
Example:

Convert $3\frac{1}{8}$ into a decimal.

$1 \div 8 = 0.125$

$0.125 + 3 = 3.125$

Some useful fraction and decimal equivalents to remember:

\[
\begin{align*}
\frac{1}{2} &= 0.5 \\
\frac{1}{3} &= 0.333\ldots \\
\frac{2}{3} &= 0.666\ldots \\
\frac{1}{4} &= 0.25 \\
\frac{3}{4} &= 0.75 \\
\frac{1}{5} &= 0.2 \\
\frac{2}{5} &= 0.4 \\
\frac{3}{5} &= 0.6 \\
\frac{4}{5} &= 0.8 \\
\frac{1}{10} &= 0.1
\end{align*}
\]

Converting Decimals to Fractions

To convert a decimal to a fraction, follow the following steps:

- Write the decimal in fraction form with a denominator of 1 and decimal in the numerator.
- Multiply both numerator and denominator by a multiple of 10 that will make the decimal a whole number.
- Reduce the fraction if you can.

Example:

Convert 0.82 into a fraction.

First, write the decimal in fraction form by writing the decimal over 1.

\[
\frac{0.82}{1}
\]

Next, multiply both the numerator and denominator by 100 since we want to get rid of two decimals places in 0.82.

\[
\frac{0.82 \times 100}{1 \times 100} = \frac{82}{100}
\]

Now, reduce the fraction.
The final answer is \( \frac{41}{50} \).

**Practice question:**

29. Which of the following fractions are equivalent to 0.05?

a) \( \frac{19}{20} \)

b) \( \frac{1}{20} \)

c) \( \frac{1}{2} \)

d) \( \frac{1}{10} \)

**Introduction to Percent**

Percent means "per hundred".

Like fractions, percents are a way to represent parts of one whole. However, in percent one whole is always considered to be 100%.

1 whole = 100%

**Converting Decimals or Fractions to Percent**

To convert a fraction into a percent, multiply by \( \frac{100}{1} \) and then simplify as much as you can (e.g. reduce the fraction, convert an improper fraction to a mixed number, etc.). Another way to convert a fraction into a percent is to convert the fraction into a decimal first and then multiply by 100.

**Example:**

Convert \( \frac{1}{3} \) into a percentage.

\[
\frac{1}{3} \times \frac{100}{1} = \frac{100}{3}
\]

Step 1: Multiply the fraction by \( \frac{100}{1} \).

\[
\frac{100}{3} = 33 \frac{1}{3}
\]

Step 2: Change the improper fraction into a mixed number.

Final answer is \( 33 \frac{1}{3} \% \).
To convert a decimal into a percent, multiply the decimal by **100**. The shortcut method with multiplying decimals by 100 is to move the decimal place over to the right two times.

**Example:**

Convert 0.028 into a percentage.

$0.028 \times 100 = 2.8\%$

**Practice question:**

30. What is $\frac{2}{5}$ expressed as a percent?

- a) $\frac{2}{5} \%$
- b) 0.4 \%
- c) 4\%
- d) 40\%

**Converting Percent to Decimals or Fractions**

To convert a percent into a fraction, write the percent as the numerator of a fraction with a denominator of 100. (This is the same as dividing the percent by 100). Simplify fully.

\[
\frac{\text{percent}}{100}
\]

**Example:**

Convert 52\% into a fraction.

\[
52\% = \frac{52}{100}
\]

Step 1: Write the percent as the numerator. Denominator is 100.

\[
\frac{52}{100} = \frac{13}{25}
\]

Step 2: Reduce the fraction.

Final answer is $\frac{13}{25}$.

To convert a percent into a decimal, divide the percent by 100. The shortcut method to divide decimals by 100 is to move the decimal place over to the left two times.
Example:

Convert 3.9% into a decimal.

3.9% ÷ 100 = 0.039

Final answer is 0.039.

Practice question:

**31. What is 34% expressed as a fraction?**

- a) $\frac{34}{50}$
- b) $\frac{34}{1}$
- c) $\frac{17}{50}$
- d) $\frac{17}{100}$

4. Percent

Common Word problems involving percent:

Examples of the three most common word problems involving percent are:

- What is 15% of 30?
- 30 is 15% of what number?
- 15 is what percent of 30?

To solve word problems in the format “What is x% of y?”, follow these steps:

1. Convert the percentage into a decimal.
2. Multiply the decimal by the number given in the question.

Solution for “What is 15% of 30?”

15% = 15 ÷ 100 = 0.15

0.15 x 30 = 4.5

Therefore, 15% of 30 is **4.5**.
To solve word problems in the format “y is x% of what number?”, follow these steps:
1. Convert the percentage into a decimal.
2. Divide the number given in the question by the decimal.

Solution for “30 is 15% of what number?”

15% = 15 ÷ 100 = 0.15
30 ÷ 0.15 = 200
Therefore, 30 is 15% of 200.

To solve word problems in the format “x is what percent of y?”, follow these steps:
1. Divide the part, x, by the whole, y.
2. Multiply the resulting quotient by 100.

Solution for “15 is what percent of 30?”

15 ÷ 30 = 0.5
0.5 x 100 = 50
Therefore, 15 is 50% of 30.

Practice questions:

32. What is 60% of 500?
   a) 3000
   b) 300
   c) 30
   d) 833.33

33. 15 is 75% of what number?
   a) 20
   b) 200
   c) 11.25
   d) 5

34. 200 is what percent of 50?
   a) 40%
   b) 25%
   c) 4%
   d) 400%
Sales Tax

Sales tax is expressed as a certain percentage of a sales price. The sales tax amount ($) is added to the selling price in order to get the final total price.

When solving for the total price of a product after taxes use the following steps:

Method 1:
1. Find the sales tax amount by converting the sales tax percentage into a decimal and then multiplying by the selling price.
2. Add the sales tax amount to the original selling price.

OR

Method 2:
1. Convert the sales tax percentage into a decimal. Then add 1. (The decimal part of this number represents the amount of tax; 1 represents the original sales price).
2. Multiply the number from the previous step by the original sales price to get the total final price.

Note: when working with dollar amounts, always round your answer to 2 decimal places.

Example:
A t-shirt costs $20.99. What is the final price of this t-shirt after a tax of 13% is added?

Method 1:
Step 1: 13% ÷ 100 = 0.13 0.13 x 20.99 = 2.73
Step 2: 20.99 + 2.73 = 23.72
Therefore, the final price of the t-shirt is $23.72.

Method 2:
Step 1: 13% ÷ 100 = 0.13
0.13 + 1 = 1.13
Step 2: 1.13 x 20.99 = 23.72
Therefore, the final price of the t-shirt is $23.72.

Practice question:
35. What is the final price of a textbook that costs $187 after 13% tax is added?

   a) 211.31  
   b) 199.99  
   c) 24.31  
   d) 1438.46

**Price Discounts**

Discount is usually expressed as a certain percentage of a selling price (e.g. 30% off). The sales tax amount ($) is *subtracted* from the selling price in order to get the final price.

---

When solving for the total price of a product after a discount has been applied use the following steps:

**Method 1:**

1. Find the discount amount by converting the discount percentage into a decimal and then multiplying by the original price.
2. Subtract the discount amount from the original price.

**OR**

**Method 2:**

1. Convert the discount percentage into a decimal. Subtract this decimal from 1. (This decimal represents what *part* of the original price is the discounted price.)
2. Multiply the number from the previous step by the original price to get the final discounted price.

---

**Example:**

A camera costs $299. Right now, the camera is on sale at 15% off. What is the discounted price of the camera?

**Method 1:**

Step 1: $15\% \div 100 = 0.15$

0.15 x 299 = 44.85

Step 2: 299 – 44.85 = 254.15

Therefore, the discounted price of the camera is $254.15.
Method 2:
Step 1: 15% ÷ 100 = 0.15
1 – 0.15 = 0.85
Step 2: 0.85 x 299 = 254.15
Therefore, the discounted price of the camera is $254.15.

Practice question:
36. A box of pens costs $17.99. The pens are currently on sale at 30% off. What is the discounted price of a box of pens?
   a) 14.99  
   b) 59.97  
   c) 5.40   
   d) 12.59

Percent Increase

Questions that involve a **percent increase** can be solved using the same strategies as for prices after taxes. Taxes are really just a percent increase.

To find the number **after percent increase**:

Step 1. Express percent as a decimal by dividing the percent by 100.

Step 2. Multiply the decimal by the number representing the total to get the increase amount.

Step 3. Add the increase amount and the original amount.

Example:
The sales of company A totaled $389,000 in 2010. In 2011, the sales increased by 3%. What is the amount of sales for company A in 2011?

Solution:
3% ÷ 100 = 0.03
0.03 x 389000 = 11670
389000 + 11670 = 400670
Therefore, the sales in 2011 were $400,670.
Practice Question:

37. In the fall semester, a total of 780 students visited the Tutoring and Learning Centre. In the winter semester, the number of students increased by 5%. What is the number of students who attended the Tutoring and Learning Centre in the winter semester?
   a) 15600
   b) 1170
   c) 39
   d) 819

Percent Decrease

Questions that involve a percent decrease can be solved using the same strategies as for prices after discount. Discounts are really just a percent decrease.

To find the number after percent decrease:

Step 1. Express percent as a decimal by dividing the percent by 100.

Step 2. Multiply the decimal by the number representing the total to get the decrease amount.

Step 3. Subtract the decrease amount from the original amount.

Example:

The population of a small northern town was 6780 people. This year, the population decreased by 3.2%. What is the population of the town this year?

Solution:

3.2% ÷ 100 = 0.032

0.032 x 6780 = 216.96 = 217 (We cannot have 0.96 of a person. Therefore, round the answer to the nearest whole number.)

6780 – 217 = 6563

Therefore, the population of the town this year is 6583 people.

Practice Question:

38. Last year there were 3180 cases of the flu treated at hospital A. This year, the number of cases of flu decreased by 12.7%. What is the number of cases of flu for this year?
   a) 404
   b) 2776
Finding the Percent in Percent Increase or Percent Decrease.

When finding the percent by which a number has increased or decreased, use the following guidelines:

Find the difference between the original number and the number after increase/decrease

1. Divide the difference by the original number. (You should get a decimal answer.)
2. Multiply the decimal by 100 to get a percentage.

Note: The percent can be larger than 100%.

Examples:

Problem 1:
The average price of a detached home in city A was $506,000 in 2010. By 2012, the average price increased to $745,000. By what percentage has the average home price increased from 2010 to 2012? (State the final answer to 2 decimal places.)

Solution:

$745000 – 506000 = 239000$

$239000 ÷ 506000 = 0.4723$

$0.4723 \times 100 = 47.23\%$

Therefore, the average home price increased by 47.23% from 2010 to 2012.

Problem 2:

In 1990 the number of families in the GTA relying on social assistance was 35700. By 2000 this number decreased to 32100. By what percentage did the number of families in the GTA relying on social assistance decrease between 1990 and 2000? (State the final answer to the nearest whole percent.)

Solution:

$35700 – 32100 = 3600$

$3600 ÷ 35700 = 0.10$

$0.10 \times 100 = 10\%$
Therefore, there was a 10% decrease in the number of families in the GTA relying on social assistance between 1990 and 2000.

Practice Questions:

39. Due to inflation, the price of food has increased. If a loaf of bread cost $1.99 five years ago and costs $2.49 today, what is the percent increase in the price of a loaf of bread?
   a) 20%
   b) 25%
   c) 10%
   d) 80%

40. A company's profit was $897,000 in 2010. Following recession, the company's profit decreased to $546,000 in 2011. What was the percent decrease in the company's profit between 2010 and 2011?
   a) 64%
   b) 27%
   c) 13%
   d) 39%

6. Rates

A rate is a ratio that compares two quantities with different units.

For example, your cell phone company might charge you 35¢ per minute for a long distance call. 35¢ per minute is a rate that compares cost in cents to time in minutes.

A unit rate is a special type of rate where the first type of quantity corresponds to one unit of the second type of quantity.

The long distance charge example from above, 35¢ per minute, is also an example of a unit rate since we are comparing cost to one unit of time (one minute).

Other examples of common unit rates:

- kilometers per hour (e.g. 100 km/h)
- cost per pound (e.g. $3.75/lb)
- earnings per hour (e.g. $10.25/h)
Any rate can be converted to a *unit* rate by dividing the first quantity by the second quantity.

**Example 1:**
A local grocery store sells raisins at $6 for 500 grams. What is the unit price of the raisins?

Solution:
$6 ÷ 500 \text{ g} = 0.012/\text{g}$

Therefore, the unit price of the raisins is $0.012/\text{g}.$

**Example 2:**
It takes 30 hours to fill a 100 liters tank with water. At the same rate, how long will it take to fill a 120 L tank with water?

Solution (using unit rate):
First we find the unit rate for filling a tank with water.
$30 \text{ hrs} ÷ 100 \text{ L} = 0.3 \text{ hrs/L}$

Now we use the unit rate to find how long it will take to fill a 120 L tank with water.

$0.3 \text{ hrs/L} \times 120 \text{ L} = 36 \text{ hrs}$

Therefore, at the same rate, it will take 36 hours to fill a 120 L tank with water.

When working with rates, it is a good idea to include **units** in the calculations. Units can be used as a guide for doing the correct operation (e.g. multiplying or dividing). If you have done the correct calculation you will be left with the desired units in the answer.

**Example:**

A marathon runner completes a 10 km race in 0.75 hours. Assuming the same pace, how long will it take the runner to complete a full 42 km marathon?

Solution (using unit rate):
First, we find the unit rate or speed of the runner.

$10 \text{ km} ÷ 0.75 \text{ hr} = 13.33 \text{ km/hr} \ (\text{answer is rounded to two decimal places})$

Now, we use the unit rate to see how long it will take the runner complete 42 km.
Step 1: Since you want “km” to cancel out, divide distance by speed. Unit rate (speed) is written in fraction form to make the units distinct.

\[
42 \text{ km} \div \frac{13.33 \text{ km}}{1 \text{ hr}}
\]

Step 2: In order to divide, we multiply by the reciprocal. “km” in the numerator and “km” in the denominator cancel out.

\[
= 42 \text{ km} \times \frac{1 \text{ hr}}{13.33 \text{ km}}
\]

Step 3: The answer has the desired units, which is hours in this question. Answer is rounded to two decimal places.

\[
= 3.15 \text{ hr}
\]

Therefore, at the same pace, it will take the runner 3.15 hours to complete the full marathon.

Practice questions:

41. A shipping company charges $36 for a 12 kg package shipping to France. What is the unit price for shipping packages to France?
   a) $3/kg
   b) $0.33/kg
   c) $36/kg
   d) $12/kg

42. Lucy can walk from home to school, a distance of 2 km, in 0.25 hours. Assuming the same pace, how long will it take Lucy to walk from home to the mall, which is 5 km away?
   a) 0.5 hours
   b) 0.208 hours
   c) 0.625 hours
   d) 1.6 hours

7. Squares and Square Roots

Squares

To **square** a number means to **multiply the number by itself**. A square is indicated with the number two \( (2) \) as a superscript.

For example, 5 squared is written as \( 5^2 \) and is equal to \( 5 \times 5 = 25 \).

A common mistake that many students make when evaluating \( 5^2 \) is to do \( 5 \times 2 \). This is not correct since the 2 as a superscript indicates that you need to multiply 5 by itself.
The entire number, including the superscript, is referred to as a **power**. The superscript is the **exponent**. The number that is being multiplied by itself is called the **base**.

Example:

```
  6^2
  -----
  | base |
  |  exponent |
```

If the base on a power is negative, it must be included in brackets. For example, \((-8)^2\)
\((-8)^2 = (-8)(-8) = 64\)

If there are no brackets, the base is positive but the power is negative. For example, \(-8^2\)
\(-8^2 = -(8)(8) = -64\)

As you can see from the above example, \((-8)^2\) is NOT the same as \(-8^2\).

**Practice questions:**

43. Evaluate \(-11^2\)
   a) 121
   b) \(-121\)
   c) \(-22\)
   d) 22

44. Evaluate \(-(-9)^2\)
   a) \(-18\)
   b) 81
   c) 18
   d) \(-81\)

**Square Roots**

Square root, indicated by the symbol, \(\sqrt{\quad}\), is the opposite operation of squaring a number.

The square root of a number is defined below:

If \(x^2 = y\), then \(\sqrt{y} = x\)
When finding the square root of a number, we are looking for a number that multiplied by itself would give us the number underneath the square root.

Example:

Evaluate $\sqrt{16}$

What number multiplied by itself would equal to 16? One such number is 4, since $4 \times 4 = 16$. Another possible number is $-4$, since $(-4)(-4) = 16$. Thus, the square root of 16 is equal to 4 or $-4$. Both of these are valid answers.

Practice question:

45. What is the $\sqrt{121}$?
   a) 11
   b) $\frac{121}{2}$
   c) $-11$
   d) Both a and c.

From above, 16 and 121 are both examples of **perfect squares**.

Perfect squares are numbers whose square root is a whole number.

There is an infinite number of perfect squares. Some are listed below:

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, …

It is good to remember and be able to recognize which numbers are perfect squares and which are not. Knowing the perfect squares can help to estimate the value of the square root of a number that is not a perfect square.

Example:

Estimate the value of $\sqrt{62}$.

62 lies between the two perfect squares 49 and 64. Thus, the square root of 62 would be somewhere between 7 ($= \sqrt{49}$) and 8 ($= \sqrt{64}$). Since 62 is much closer to 64 than to 49, the square root of 62 would be closer to 8 than to 7. A reasonable guess might be 7.8 or 7.9.

The actual value for $\sqrt{62}$ is 7.874007874…

NOTE: Most calculators only give the positive square root of a number. Likewise, we usually only consider the positive root unless both the negative and positive roots matter in our particular problem.
It is possible to evaluate square roots of numbers that are not perfect squares by hand (not discussed here) or using a calculator. On your calculator, use the $\sqrt{}$ button to evaluate square roots.

**Practice questions:**

46. Estimate $\sqrt{93}$. *(Note: no calculator allowed.)*

   a) Between 7 and 8.
   b) Between 10 and 11.
   c) Between 9 and 10.
   d) Between 8 and 9.

47. Evaluate the $\sqrt{28}$. Round the answer to one decimal place. *(Note: use a calculator.)*

   a) 6
   b) 5
   c) 5.3
   d) None of the above

8. Basic Geometry

**Perimeter** is the **sum of the lengths of all sides** for a particular geometric shape. For a circle, circumference is the special name for the perimeter or distance around the circle.

**Area** is defined as the **number of square units that cover a closed figure**, such as a rectangle, square, parallelogram, circle, etc.

Formulas for perimeter and area of basic geometric shapes are presented in the table below.

<table>
<thead>
<tr>
<th>Name</th>
<th>Diagram</th>
<th>Perimeter Formula</th>
<th>Area Formula</th>
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</thead>
<tbody>
<tr>
<td>Rectangle</td>
<td><img src="image" alt="Rectangle Diagram" /></td>
<td>$P = 2 (l + w)$</td>
<td>$A = l \times w$</td>
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<tr>
<td>Square</td>
<td><img src="image" alt="Square Diagram" /></td>
<td>$P = 4s$</td>
<td>$A = s^2$</td>
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</tbody>
</table>

Note: $l$ is length, $w$ is width, $s$ is side length.
<table>
<thead>
<tr>
<th>Parallelogram</th>
<th>[ \text{w} ]</th>
<th>[ \text{h} ]</th>
<th>[ \text{b} ]</th>
<th>[ P = 2 (\text{b} + \text{w}) ]</th>
<th>[ A = \text{b} \times \text{h} ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Note: ( \text{w} ) is width</td>
<td>[ \text{h} ] is height</td>
<td>[ \text{b} ] is length of base</td>
<td>[ P = 2 (\text{b} + \text{w}) ]</td>
<td>[ A = \text{b} \times \text{h} ]</td>
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</table>

<table>
<thead>
<tr>
<th>Triangle</th>
<th>[ s_1 ]</th>
<th>[ s_2 ]</th>
<th>[ \text{h} ]</th>
<th>[ \text{b} ]</th>
<th>[ P = \text{b} + s_1 + s_2 ]</th>
<th>[ A = \text{b} \times \text{h} ]</th>
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</thead>
<tbody>
<tr>
<td>Note: ( s_1 ) is length of side 1</td>
<td>[ s_2 ] is length of side 2</td>
<td>[ \text{h} ] is height</td>
<td>[ \text{b} ] is length of base</td>
<td>[ P = \text{b} + s_1 + s_2 ]</td>
<td>[ A = \frac{\text{b} \times \text{h}}{2} ]</td>
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</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Circle</th>
<th>[ r ]</th>
<th>[ C = 2\pi r ]</th>
<th>[ A = \pi r^2 ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Note: ( r ) is radius</td>
<td>[ C = 2\pi r ] (\text{where } \pi \approx 3.14)</td>
<td>[ A = \pi r^2 ]</td>
<td></td>
</tr>
</tbody>
</table>

**Example:**

Find the area of a rectangle whose width is 4.5 cm and length is twice the width.

**Solution:**

- \( w = 4.5 \text{ cm} \)
- \( l = 2 \times 4.5 \text{ cm} \)
- \( l = 9 \text{ cm} \)
- \( A = l \times w \)
- \( A = 4.5 \text{ cm} \times 9 \text{ cm} \)
- \( A = 40.5 \text{ cm}^2 \)

**Practice questions:**

48. Find the area of a square whose perimeter is 12 cm.
   - a) 9 cm²
   - b) 16 cm²
   - c) 144 cm²
   - d) 36 cm²
49. Find the area of a circle with radius 5.3 cm.
   a) 16.64 cm²
   b) 28.09 cm²
   c) 88.20 cm²
   d) 33.28 cm²
Appendix A: Glossary

**Area** – the number of square units an enclosed figure covers

**Circumference** – perimeter of a circle; the distance around a circle

**Decimal** – a number that uses a decimal point followed by digits that expresses values less than one; the fractional part of a decimal is based on powers of 10.

**Decimal point** – a dot or point used to separate the whole number from the fractional part in a decimal

**Denominator** – the “bottom” number in a fraction; the denominator indicates the number of parts into which one whole is divided.

**Difference** – the result of subtraction

**Dividend** – the number that is being divided

**Divisor** – the number that a dividend is divided by

**Equivalent fractions** – fractions that are equal in value; on a number line, equivalent fractions would occupy the same spot.

**Estimate** – to give a reasonable guess

**Evaluate** – to calculate the numerical value

**Exponent** – the number in a power indicating how many times repeated multiplication is done

**Factor** – a number that will divide into another number exactly; (For example, factors of 6 are 1, 2, 3, and 6).

**Greatest common factor** – the largest number that is a factor of two or more given numbers; (For example, the greatest common factor of 27 and 36 is 9).

**Improper fraction** – a fraction where the numerator is greater than the denominator; An improper fraction is always greater than one whole.

**Lowest common denominator** – the lowest common multiple of the fractions’ denominators

**Lowest common multiple** – the smallest number that is common in sets of multiples for two or more numbers; (For example, the lowest common multiple of 3 and 4 is 12).

**Lowest terms** – a fraction is considered to be in lowest terms when it cannot be reduced further; this occurs where there are no more common factors of the numerator and denominator other than 1.

**Mixed number** – a number consisting of a whole number and a fraction
Multiple – the product of a number and any whole number; (For example, multiples of 3 are 3, 6, 9, 12, 15, 18, …)

Numerator – the “top” number in a fraction; the numerator indicates how many parts of a whole the fraction represents

Percent – parts per 100

Perfect square – a number whose square root is a whole number

Perimeter – the total distance around an enclosed figure

Place value – the location of a digit in a number and the specific name for that location

Source: http://www.onlinemathlearning.com/place-value-chart.html

Power – a number raised to an exponent

Product – the result of multiplication

Proper fraction – a fraction where the numerator is less than the denominator; A proper fraction is always less than one whole.

Quotient – the result of division

Rate – a comparison of two quantities with different units

Ratio – a comparison of two quantities with same units

Rational expression – an expression written in fraction form, \( \frac{a}{b} \). (Note: the denominator of a rational expression cannot equal 0.)

Reciprocal – the multiplicative inverse of a number; The product of two reciprocals is by definition equal to 1. For a fraction, \( \frac{a}{b} \), the reciprocal is \( \frac{b}{a} \).

Squared – raised to the exponent 2

Square root – the value of the number, which multiplied by itself, gives the original number
**Sum** – the result of addition

**Unit rate** – a special type of rate where the first type of quantity corresponds to *one* unit of the second type of quantity; (For example, 30 km/h).
## Appendix B: Multiplication Table

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Appendix C: Rounding Numbers

Follow these steps for rounding numbers to a specified place value or decimal place.

1. Identify the directions for rounding. (E.g. Round to the nearest tenth. Round to two decimal places.)
2. Identify the number in the place value or the decimal place to be rounded to. Underline this number.
3. Look at the number to the right of the underlined number. (Note: Only look at ONE number immediately to the right.)
4. If the number to the right is 5 or higher, increase the underlined number by 1. If the number to the right is 4 or lower, keep the underlined number the same. (Do NOT decrease the number).
5. If the underlined number is in the decimal place values, delete all numbers to the right of the underlined number.
   If the underlined number is in the whole number place values, replace all numbers up to the decimal point with zeroes.

Example:

Round 5.8739 to two decimal places.

5.8739

The number to the right of 7 is 3.
3 is less than 5; therefore, keep 7 as is.
Delete all numbers to the right of 7.
5.87 Final answer.

Example 2:

Round 14590 to the nearest thousand.

14590

The number to the right of 4 is 5. Therefore, increase 4 by 1.
4 + 1 = 5.
Replace all numbers to the right of 4 with zeroes.
15000 Final answer.
6. **Special case:** If the underlined number is a 9 and the number to the right is 5 or higher, the 9 would be rounded up to 10. In this case, the 1 is regrouped with the next number to the left of the underlined digit, and the underlined number is replaced with a 0.

**Example:**

Round 34.97 to one decimal place.

34.97

One decimal place

34.97 The number to the right of 9 is 7. Therefore, increase 9 by 1.
9 + 1 = 10.
Regroup 1 with the next number to the left of 9, which is 4 in this case; replace 9 with a 0.
Delete all numbers to the right of one decimal place.
35.0 Final answer.
Appendix D: Answers to Practice Questions

<table>
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<tr>
<th>Topic</th>
<th>Page Number</th>
<th>Question Number</th>
<th>Answer</th>
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