## Trigonometric Functions

This worksheet covers the basic characteristics of the sine, cosine, tangent, cotangent, secant, and cosecant trigonometric functions.

Sine Function: $f(x)=\sin (x)$

- Graph

- Domain: all real numbers
- Range: $[-1,1]$
- Period $=2 \pi$
- $x$ intercepts: $x=k \pi$, where $k$ is an integer.
- $y$ intercepts: $y=0$
- Maximum points: $(\pi / 2+2 k \pi, 1)$, where $k$ is an integer.
- Minimum points: $(3 \pi / 2+2 k \pi,-1)$, where $k$ is an integer.
- Symmetry: since $\sin (-x)=-\sin (x)$ then $\sin (x)$ is an odd function and its graph is symmetric with respect to the origin $(0,0)$.
- Intervals of increase/decrease: over one period and from 0 to $2 \pi, \sin (x)$ is increasing on the intervals $(0, \pi / 2)$ and ( $3 \pi / 2,2 \pi$ ), and decreasing on the interval ( $\pi / 2,3 \pi / 2$ ).

Cosine Function: $f(x)=\cos (x)$

- Graph

- Domain: all real numbers
- Range: $[-1,1]$
- Period $=2 \pi$
- $x$ intercepts: $x=\pi / 2+k \pi$, where $k$ is an integer.
- y intercepts: $y=1$
- Maximum points: $(2 k \pi, 1)$, where $k$ is an integer.
- Minimum points: $(\pi+2 k \pi,-1)$, where $k$ is an integer.
- Symmetry: since $\cos (-x)=\cos (x)$ then $\cos (x)$ is an even function and its graph is symmetric with respect to the $y$ axis.
- Intervals of increase/decrease: over one period and from 0 to $2 \pi, \cos (x)$ is decreasing on $(0, \pi)$ increasing on $(\pi, 2 \pi)$.


## Trigonometric Functions

Tangent Function : $f(x)=\tan (x)$

- Graph

- Domain: all real numbers except $\pi / 2+k \pi, k$ is an integer.
- Range: all real numbers
- Period $=\pi$
- $\quad x$ intercepts: $x=k \pi$, where $k$ is an integer.
- $y$ intercepts: $y=0$
- Symmetry: since $\tan (-x)=-\tan (x)$ then $\tan (x)$ is an odd function and its graph is symmetric with respect the origin.
- Intervals of increase/decrease: over one period and from $-\pi / 2$ to $\pi / 2$, $\tan (x)$ is increasing.
- Vertical asymptotes: $x=\pi / 2+k \pi$, where $k$ is an integer.


## Trigonometric Functions

Cotangent Function : $f(x)=\cot (x)$

- Graph

- Domain: all real numbers except $\mathrm{k} \pi, \mathrm{k}$ is an integer.
- Range: all real numbers
- Period $=\pi$
- $\quad x$ intercepts: $x=\pi / 2+k \pi$, where $k$ is an integer.
- Symmetry: since $\cot (-x)=-\cot (x)$ then $\cot (x)$ is an odd function and its graph is symmetric with respect the origin.
- Intervals of increase/decrease: over one period and from 0 to $\pi, \cot (x)$ is decreasing.
- Vertical asymptotes: $\mathrm{x}=\mathrm{k} \pi$, where k is an integer.

Secant Function: $f(x)=\sec (x)$

- Graph

- Domain: all real numbers except $\pi / 2+\mathrm{k} \pi, \mathrm{n}$ is an integer.
- Range: (-ve infinity , -1] $\cup[1$, +ve infinity)
- Period $=2 \pi$
- y intercepts: $y=1$
- Symmetry: since $\sec (-x)=\sec (x)$ then $\sec (x)$ is an even function and its graph is symmetric with respect to the $y$ axis.
- Intervals of increase/decrease: over one period and from 0 to $2 \pi, \sec (x)$ is increasing on $(0, \pi / 2) \cup(\pi / 2, \pi)$ and decreasing on $(\pi, 3 \pi / 2) \cup(3 \pi / 2,2 \pi)$.
- Vertical asymptotes: $x=\pi / 2+k \pi$, where $k$ is an integer.


## Trigonometric Functions

## Cosecant Function: $f(x)=\csc (x)$

- Graph

- Domain: all real numbers except $\mathrm{k} \pi, \mathrm{k}$ is an integer.
- Range: (-ve infinity , -1$] \cup[1$, +ve infinity)
- Period $=2 \pi$
- Symmetry: since $\csc (-x)=-\csc (x)$ then $\csc (x)$ is an odd function and its graph is symmetric with respect to the origin.
- Intervals of increase/decrease: over one period and from 0 to $2 \pi, \csc (x)$ is decreasing on $(0, \pi / 2) \cup(3 \pi / 2,2 \pi)$ and increasing on $(\pi / 2, \pi) \cup(\pi / 3 \pi / 2)$.
- Vertical asymptotes: $x=k \pi$, where $k$ is an integer.

