Reference Angle

If A is an angle in standard position, its reference angle, $A_r$, is the acute angle formed by the x-axis and the terminal side of angle A.

If angle A is in quadrant II then the reference angle

\\[ A_r = 180^\circ - A \]

(if A is given degrees)

or

\\[ A_r = \pi - A \]

(if A is given in radians).

If angle A is in quadrant I then the reference angle

\\[ A_r = A. \]

If angle A is in quadrant III then the reference angle

\\[ A_r = A - 180^\circ \]

(if A is given degrees)

or

\\[ A_r = A - \pi \]

(if A is given in radians).

If angle A is in quadrant IV then the reference angle

\\[ A_r = 360^\circ - A \]

(if A is given degrees)

or

\\[ A_r = 2\pi - A \]

(if A is given in radians).
Reference Angle

Example

1. Find the reference angle for each of the following angles.

   a) $A = 120^\circ$
   b) $A = -\frac{15\pi}{4}$
   c) $A = -30^\circ$

Solutions

a) Since angle A is in quadrant II, the reference angle, $A_r = 180^\circ - 120^\circ = 60^\circ$

b) The given angle is not positive and less than $2\pi$. We can use the positive and less than $2\pi$ coterminal $A_c$ to angle A. $A_c = -\frac{15\pi}{4} + 2(2\pi) = \frac{\pi}{4}$

d) Angle A is negative, in quadrant IV and its absolute value is less than $90^\circ$. Hence $A_r = | -30^\circ | = 30^\circ$
Reference Angle

Practice Question

1. Find the reference angle for each of the following angles.

   a) \( A = 1620^\circ \)
   b) \( A = -29\pi/7 \)
   c) \( A = -\pi/7 \)

Answers

1. a) \( A_r = 25^\circ \)
   b) \( A_r = \pi/6 \)
   c) \( A_r = \pi/7 \)