

# Reference Angle

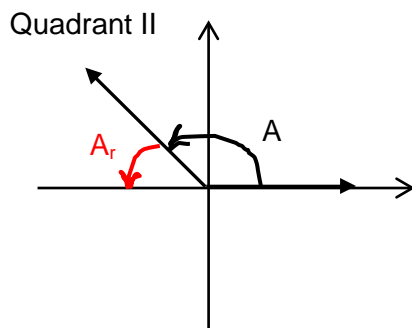
If  $A$  is an angle in standard position, its **reference angle,  $A_r$** , is the **acute angle formed by the x-axis and the terminal side of angle  $A$** .

If angle  $A$  is in quadrant II then the reference angle

$$A_r = 180^\circ - A$$

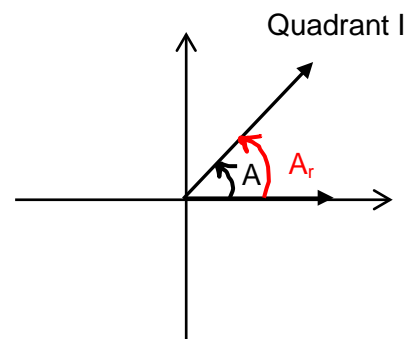
(if  $A$  is given degrees)  
or  
$$A_r = \pi - A$$

(if  $A$  is given in radians).



If angle  $A$  is in quadrant I then the reference angle

$$A_r = A.$$

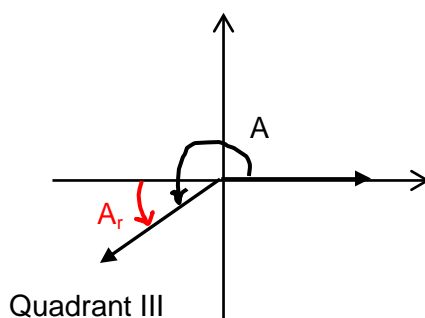


If angle  $A$  is in quadrant III then the reference angle

$$A_r = A - 180^\circ$$

(if  $A$  is given degrees)  
or  
$$A_r = A - \pi$$

(if  $A$  is given in radians)

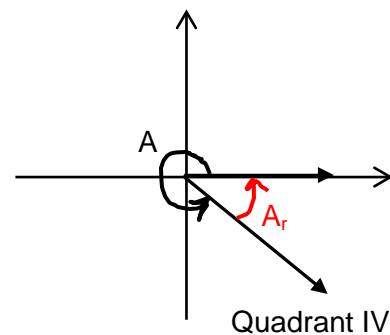


If angle  $A$  is in quadrant IV then the reference angle

$$A_r = 360^\circ - A$$

(if  $A$  is given degrees)  
or  
$$A_r = 2\pi - A$$

(if  $A$  is given in radians)



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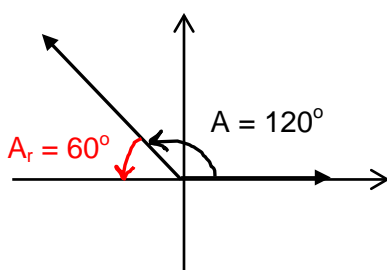
## Example

1. Find the reference angle for each of the following angles.

- a)  $A = 120^\circ$
- b)  $A = -15\pi/4$
- c)  $A = -30^\circ$

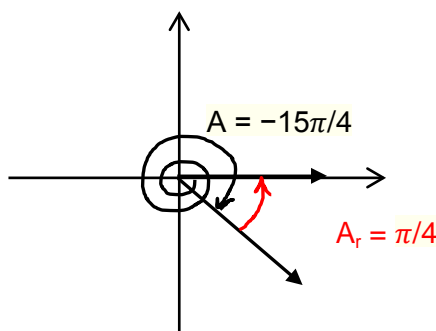
## Solutions

a) Since angle  $A$  is in quadrant II, the reference angle,  $A_r = 180^\circ - 120^\circ = 60^\circ$



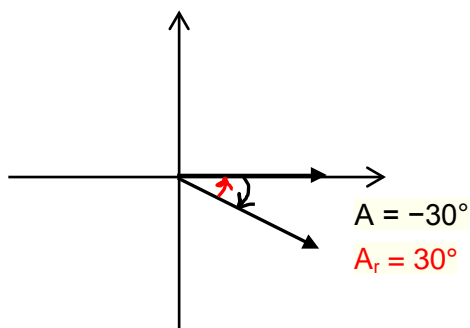
b) The given angle is not positive and less than  $2\pi$ . We can use the positive and less than  $2\pi$  coterminal  $A_c$  to angle  $A$ .

$$A_c = -15\pi/4 + 2(2\pi) = \pi/4$$



d) Angle  $A$  is negative, in quadrant IV and its absolute value is less than  $90^\circ$ . Hence

$$A_r = | -30^\circ | = 30^\circ$$



# Reference Angle

## Practice Question

1. Find the reference angle for each of the following angles.

a)  $A = 1620^\circ$

b)  $A = -29\pi/7$

c)  $A = -\pi/7$

## Answers

1. a)  $A_r = 25^\circ$

b)  $A_r = \pi/6$

c)  $A_r = \pi/7$