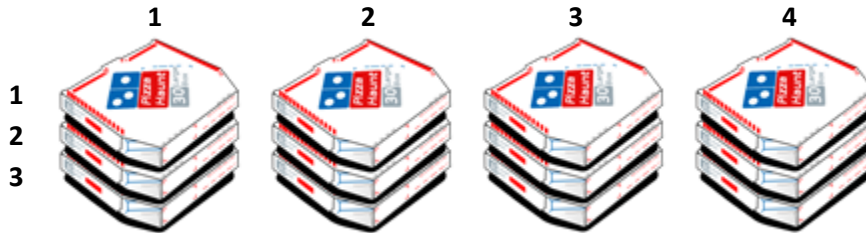


Multiplying and Dividing Fractions

When multiplying whole numbers we are counting a repeated set of items.

Exercise 1: How many pizza boxes are there?



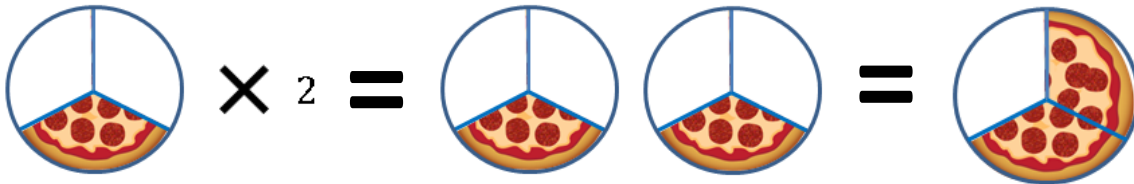
There are 3 rows and 4 columns of boxes. Thus, we have $3 \times 4 = 12$ pizza boxes.

Part A – Multiplication of Fractions

Fractions can be multiplied by a whole number or by another fraction. Thus, when multiplying fractions we are **counting a repeated set of parts (Case 1)** OR **taking a part of a part (Case 2)**.

Case 1: Fractions Multiplied by a Whole Number.

Exercise 2: Multiply the fractions, $\frac{1}{3} \times 2$.

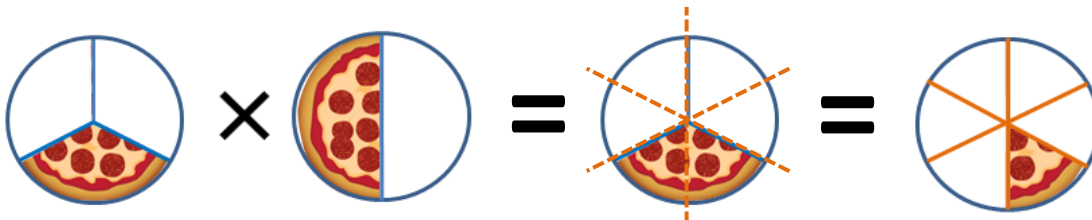


Since there are two multiples of $\frac{1}{3}$, we have $\frac{1}{3} \times 2 = \frac{2}{3}$.

Case 2: Fractions Multiplied by a Fraction.

Exercise 3: Multiply the fractions, $\frac{1}{3} \times \frac{1}{2}$.

Note: $\frac{1}{3} \times \frac{1}{2}$ is the same as $\frac{1}{2}$ of $\frac{1}{3}$. Thus, we are cutting each third in half.



Leaving us with, $\frac{1}{3} \times \frac{1}{2} = \frac{1}{6}$ of the pizza.

Multiplying and Dividing Fractions

How does the math work?

Multiplication of Fractions

Step 1: Check! Can the fractions be reduced before multiplying? If possible, reduce by dividing the numerator and denominator of each fraction by its Greatest Common Factor (GCF) **OR** by the Cancellation Method.

Step 2: Multiply the numerator by the numerator and the denominator by the denominator.

Note: Every whole number can be written as a fraction over 1 (i.e. $2 = \frac{2}{1}$).

Exercise 4: Multiply the fractions, $\frac{4}{5} \times \frac{1}{3}$.

Step 1: Check! Since the GCF (4, 5) = 1 and the GCF (1, 3) = 1, each fraction is in its simplest form.

Step 2: Multiply the numerator by the numerator and the denominator by the denominator.

$$\begin{aligned}
 &= \frac{4}{5} \times \frac{1}{3} \\
 &= \frac{4 \times 1}{5 \times 3} \\
 &= \frac{4}{15}
 \end{aligned}$$

Exercise 5: Multiply the fractions, $\frac{13}{11} \times \frac{121}{13}$.

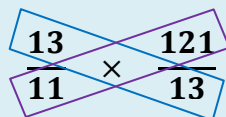
Step 1: Check! Since the GCF (13, 11) = 1 and the GCF (121, 13) = 1, each fraction is in its simplest form.

BUT WAIT! We can use the **Cancellation Method!**

Cancellation Method

The cancellation method is used before multiplying (or dividing) to avoid large numbers.

Before multiplying, we can ALSO divide the numbers **diagonally** by their **GCF**.



$$\frac{13}{11} \times \frac{121}{13}$$

Multiplying and Dividing Fractions

Step 1: Find the GCF between (13, 13) = 13. Find the GCF between (11, 121) = 11.

We can reduce both sets of numbers by dividing by their respective GCF.

Step 2: Divide each diagonal by its GCF. GCF (13, 13) = 13. GCF (11, 121) = 11

$$\begin{aligned}
 & \frac{13}{11} \times \frac{121}{13} && 13 \div 13 = 1 \\
 & && 13 \div 13 = 1 \\
 & = \frac{1}{11} \times \frac{121}{1} \\
 & = \frac{1}{11} \times \frac{121}{1} && 121 \div 11 = 11 \\
 & && 11 \div 11 = 1 \\
 & = \frac{1}{1} \times \frac{11}{1}
 \end{aligned}$$

Step 3: Now that we've reduced our fractions as much as possible, we can follow through with multiplication.

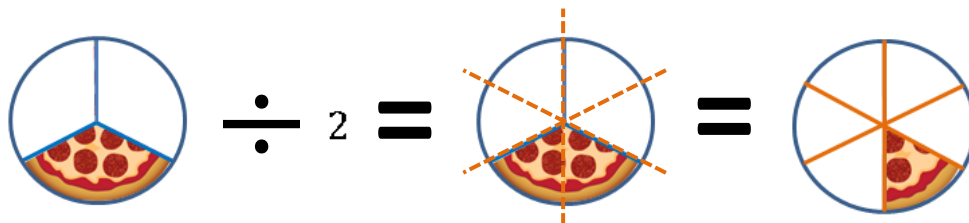
$$\begin{aligned}
 & = \frac{1 \times 11}{1 \times 1} \\
 & = \frac{11}{1} \\
 & = 11
 \end{aligned}$$

Note: The Cancellation Method only works when multiplying and dividing fractions.

Part B - Division of Fractions

When dividing fractions, we are splitting each piece into smaller, equal sized slices.

Exercise 6: Divide the fraction, $\frac{1}{3} \div 2$.



Since we are dividing each third in half, we end up with, $\frac{1}{6}$. Notice the solution for Exercise 3 is also $\frac{1}{6}$.

Multiplying and Dividing Fractions

The solutions for Exercise 3 and 6 are equal because division of fractions is actually multiplication of fractions in disguise.

Division of Fractions – Multiplication in disguise

Step 1: Flip the numerator and denominator of the second fraction.

Step 2: Replace the division sign with a multiplication sign.

Step 3: Check! Can the fractions be reduced?

Step 4: Perform multiplication.

Exercise 7: Divide the fractions, $\frac{1}{3} \div \frac{1}{2}$.

$$\begin{aligned} \frac{1}{3} \div \frac{1}{2} & \quad \curvearrowright \\ &= \frac{1}{3} \times \frac{2}{1} \\ &= \frac{1 \times 2}{3 \times 1} \\ &= \frac{2}{3} \end{aligned}$$

Step 1: Flip the numerator and denominator of the second fraction.

Step 2: Replace the division sign with a multiplication sign.

Step 3: Perform multiplication.

Exercise 8: Divide the fractions, $\frac{1}{3} \div \frac{2}{6}$.

$$\begin{aligned} \frac{1}{3} \div \frac{2}{6} & \quad \curvearrowright \\ &= \frac{1}{3} \times \frac{6}{2} \\ &= \frac{1}{1} \times \frac{2}{2} \\ &= 1 \end{aligned}$$

Step 1: Flip the numerator and denominator of the second fraction.

Step 2: Replace the division sign with a multiplication sign.

Step 3: Use the Cancellation Method.

$$\text{GCF}(6, 3) = 3$$

$$6 \div 3 = 2 \text{ and } 3 \div 3 = 1.$$

Step 4: Perform multiplication.

Multiplying and Dividing by Zero

Multiplying a whole number or fraction by zero will result in zero.

$$\frac{9}{5} \times 0 = 0$$

Multiplying and Dividing Fractions

Furthermore, division by zero is impossible since you cannot cut a whole into zero sections or parts.

Multiplying Mixed Numbers

Every mixed number must be converted to an improper fraction before multiplying or dividing. Once all the fractions are in improper form, ordinary multiplication or division can be performed.

Exercise 9: Multiply the fractions, $1\frac{1}{2} \times 4\frac{4}{7}$

$$\begin{aligned}
 & 1\frac{1}{2} \times 4\frac{4}{7} \\
 & = \frac{3}{2} \times \frac{32}{7} \\
 & = \frac{3}{1} \times \frac{16}{7} \\
 & = \frac{48}{7} \\
 & = 6\frac{6}{7}
 \end{aligned}$$

Step 1: Convert both mixed numbers into improper fractions.

$$\frac{(1 \times 2) + 1}{2} = \frac{3}{2} \qquad \frac{(4 \times 7) + 4}{7} = \frac{32}{7}$$

Step 2: Check! Can the fractions be reduced? Use the Cancellation Method.

$$\text{GCF}(32, 2) = 2$$

$$32 \div 2 = 16 \text{ and } 2 \div 2 = 1.$$

Step 3: Perform multiplication.

Step 4: Convert improper fraction into a mixed number.

Exercises:

1. Multiply or divide the following fractions.

a) $\frac{1}{3} \times \frac{4}{5}$

b) $\frac{3}{2} \times \frac{2}{3}$

c) $\frac{3}{2} \div \frac{3}{2}$

d) $\frac{5}{6} \times \frac{9}{2}$

e) $\frac{7}{3} \div \frac{1}{9}$

f) $\frac{2}{1} \times \frac{7}{6}$

g) $3 \times \frac{9}{3}$

h) $4 \times \frac{7}{11}$

i) $\frac{1}{2} \times \frac{3}{4}$

j) $\frac{0}{0} \times 0$

k) $\frac{2}{2} \times \frac{3}{4}$

l) $\frac{2}{3} \div \frac{4}{12}$

2. Evaluate the following improper fractions and mixed numbers.

a) $3\frac{1}{2} \times \frac{7}{5}$

b) $\frac{4}{3} \div \frac{4}{3}$

c) $\frac{7}{6} \div 7\frac{2}{4}$

d) $5\frac{3}{11} \div 3\frac{1}{11}$

e) $0 \div \frac{8}{3}$

f) $\frac{4}{5} \div 0$

g) $\frac{10}{9} \times \frac{3}{10}$

h) $2 \times \frac{7}{4}$

Multiplying and Dividing Fractions

i) $4\frac{4}{5} \times 8\frac{1}{3}$

j) $5\frac{0}{3} \times 1\frac{1}{5}$

k) $\frac{1}{3} \div 3\frac{1}{6}$

l) $5\frac{1}{5} \times \frac{7}{13}$

3. Evaluate the following fractions.

a) $\frac{1}{2} \times \frac{1}{3} \times \frac{1}{4}$

b) $\frac{1}{2} \div \frac{1}{3} \times 2$

c) $\frac{3}{5} \div \frac{9}{2} \times \frac{5}{3}$

d) $1\frac{3}{8} \times \frac{6}{7} \div \frac{11}{2}$

e) $3\frac{3}{19} \times 0 \div \frac{67}{180}$

f) $5\frac{1}{3} \times 0 \div 0$

g) $\frac{7}{3} \div \frac{14}{9} \times 1\frac{1}{3}$

h) $\frac{6}{3} \div \frac{1}{9} \times 2\frac{1}{3}$

Solutions:

1. Multiply or divide the following fractions.

a) $\frac{4}{15}$

b) **1**

c) **1**

d) $3\frac{3}{4}$

e) **21**

f) $2\frac{1}{3}$

g) **9**

h) $2\frac{6}{11}$

i) $\frac{3}{8}$

j) **0**

k) $\frac{3}{4}$

l) **2**

2. Evaluate the following improper fractions and mixed numbers.

a) $3\frac{1}{2}$

b) **1**

c) $\frac{14}{90}$

d) $1\frac{12}{17}$

e) **0**

f) **0**

g) $\frac{1}{3}$

h) $3\frac{1}{2}$

i) **15**

j) **18**

k) $\frac{2}{19}$

l) $2\frac{4}{5}$

3. Evaluate the following fractions.

a) $\frac{1}{24}$

b) **3**

c) $\frac{2}{9}$

d) $\frac{3}{14}$

e) **0**

f) **0**

g) **2**

h) **42**