Improper and Mixed Fractions

Fractions can be written in **proper, improper and mixed forms**. In this handout we will define each form and describe how to convert between them.

Part A – Proper Fractions

Exercise 1: Express the left over pizza slices as a fraction of the whole.

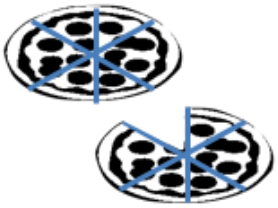
There are 4 slices left over and each whole pizza contains 6 slices. Thus, our fraction looks like

$\rightarrow \frac{4}{6}$

Definition: In a **proper fraction,** the numerator is smaller than the denominator.

Part B – Improper Fractions

Exercise 2: Express the left over pizza slices as a fraction of the whole.



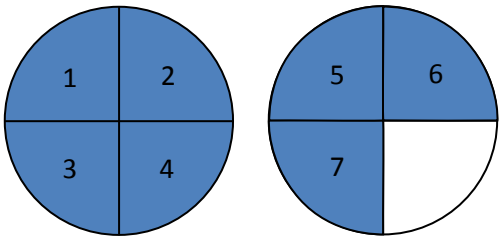
There are 11 slices left over and each whole pizza contains 6 slices. Thus, our fraction looks like

$\rightarrow \frac{11}{6}$

Improper Fractions

Definition: An **improper fraction** is when the **numerator is equal to or larger than the denominator**.

This happens when we have at least one whole AND a part of a whole.



How to create an IMPROPER FRACTION:

$$\frac{\text{Numerator}}{\text{Denominator}} = \frac{\text{Total Number of Slices}}{\text{Number of Slices in 1 Whole}} = \frac{7}{4}$$

Part C – Mixed Numbers

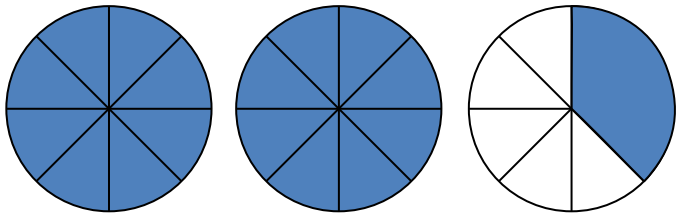
In Exercise 2 we counted and described the left over pizza as an **improper fraction**. In this next part we will see that we are also able to count and describe the left over pizza

as a **mixed number**.

Mixed Numbers

Definition: A **mixed number** is when we have a **whole number** AND a **proper fraction** combined.

Whole Number → $2\ \frac{3}{8}$ ← Proper Fraction



How to create a MIXED NUMBER:

Number of Wholes + Proper Fraction

$= 2\ wholes + \frac{3}{8}$

$= 2\ \frac{3}{8}$

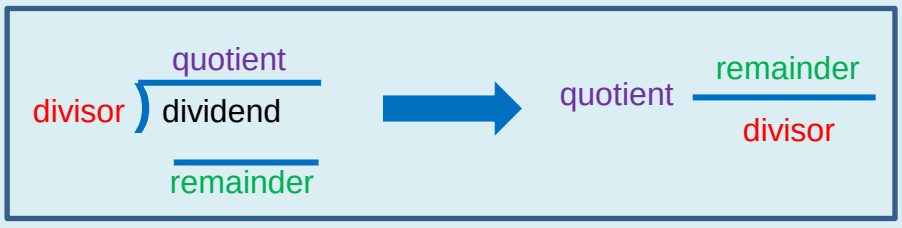
Note: Whether working with improper fractions of mixed numbers, the denominator of a fraction is ALWAYS equal to the number of slices in 1 whole.

Part D – Converting Between Improper Fractions and Mixed Numbers

To convert from an improper fraction to a mixed number follow the steps below:

Step 1: Rewrite the **fraction** as a **division problem** between the numerator and denominator.
Step 2: Perform long division.
Step 3: Rewrite the long division problem as a mixed number by:

a) setting the **quotient** equal to the **whole number,**
b) the **remainder** equal to the **numerator of the proper fraction,** and
c) the **divisor** equal to its **denominator.**



Exercise 3: Convert the improper fraction, $\frac{5}{3}$ into a mixed number.

Step 1: Rewrite the fraction as a division problem.

$\frac{5}{3} = 5 \div 3$.

Step 2: Perform long division. Remember the first number goes under the house.

$$\begin{array}{r} 1 \\ 3\overline{)\,5} \\ -\,3 \\ \hline 2 \end{array}$$

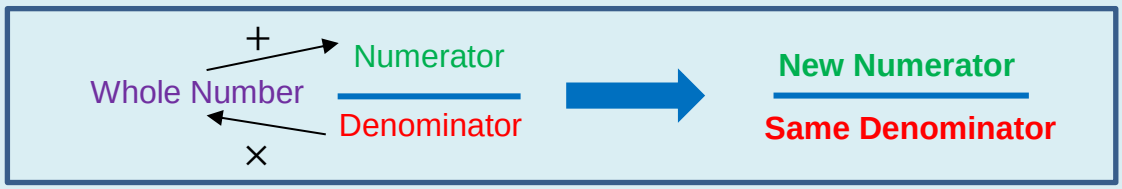
Step 3: Rewrite the long division problem as a mixed number.

$$1\ \frac{2}{3}$$

Part E – Converting Between Mixed Numbers and Improper Fractions

To convert from a mixed number to an improper fraction follow the steps below:

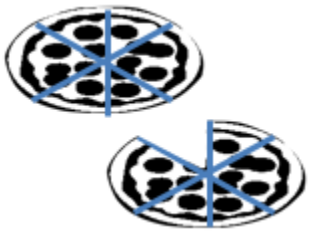
Step 1: To find the new numerator, multiply the **denominator** of the fraction by the number of **wholes** and **add** it to the **numerator**.
Step 2: The **denominator** stays the **same.**
Step 3: Rewrite as an improper fraction.



BUT how does this work?

Remember **Exercise 2?**

As a mixed number, we have → $1\frac{5}{6}$ pizzas.



To convert this mixed number to an improper fraction we need to know the total number of pizza slices.

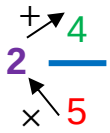
From the first pizza, we have 1 whole x 6 slices = 6 slices.
From the second pizza, we have 5 slices.

In total we have, (1 x 6) + 5 = 11 slices.

Thus, our new fraction is $\frac{11}{6}$.

Exercise 4: Convert $2\frac{4}{5}$ into a mixed fraction.

Step 1: To find the new numerator, multiply the **denominator** of the fraction by the number of **wholes** and **add** it to the **numerator**.



Step 2: The **denominator** stays the **same.**

Step 3: Rewrite as an improper fraction.

$$\frac{14}{5}$$

Simplifying Before Beginning the Question

Sometimes it is helpful to reduce fractions into their lowest terms before converting them between forms.

Exercise 5: Convert $\frac{100}{32}$ into a mixed number.

Step 1: Find the Greatest Common Factor (GCF) between 32 and 8.

100: 1, 2, **4**, 5, 10, 20, 25, 50, 100

32: 1, 2, **4**, 8, 16, 32

Step 2: Divide the numerator and denominator by the GCF.

$$\frac{100}{32} \begin{matrix} \div \mathbf{4} \\ \div \mathbf{4} \end{matrix}$$
$$= \frac{25}{8}$$

Step 3: Convert the improper fraction into a mixed number by performing long division.

$$\frac{25}{8} = 25 \div 8$$
$$= 3\frac{1}{8}$$

Improper and Mixed Fractions

Exercises:

1. Convert the following improper fractions into mixed numbers:

a) $\frac{13}{6}$ b) $\frac{35}{7}$ c) $\frac{54}{7}$ d) $\frac{19}{3}$

e) $\frac{11}{4}$ f) $\frac{8}{3}$ g) $\frac{22}{5}$ h) $\frac{61}{9}$

i) $\frac{9}{9}$ j) $\frac{41}{10}$ k) $\frac{0}{0}$ l) $\frac{16}{3}$

2. Convert the following mixed numbers into improper fractions:

a) $7\frac{3}{4}$ b) $5\frac{1}{2}$ c) $1\frac{1}{7}$ d) $5\frac{2}{9}$

e) $6\frac{4}{5}$ f) $9\frac{1}{3}$ g) $2\frac{6}{7}$ h) $1\frac{3}{7}$

i) $3\frac{0}{6}$ j) $7\frac{2}{5}$ k) $8\frac{6}{9}$ l) $10\frac{4}{8}$

3. Convert the following mixed numbers into improper fractions and vice versa: (Hint: Simplify before beginning the question.)

a) $\frac{250}{9}$ b) $\frac{70}{48}$ c) $4\frac{20}{26}$ d) $3\frac{8}{20}$

e) $1\frac{14}{24}$ f) $\frac{25}{10}$ g) $2\frac{36}{48}$ h) $\frac{56}{19}$

4. Are the following fractions equal? Justify your answers using diagrams, words, or numbers.

a) $0\frac{1}{7} = \frac{1}{7}$ b) $0\frac{0}{0} = 0$ c) $2\frac{3}{5} = 2\frac{6}{10}$ d) $4\frac{0}{9} = 4$

5. Explain using words and pictures, why you can never divide by 0?

6. Simplify the following fraction, $3\frac{6}{3}$.

Improper and Mixed Fractions

Solutions:

1. Convert the following improper fractions into mixed numbers:

a) $2\frac{1}{6}$ **b)** 5 **c)** $7\frac{5}{7}$ **d)** $6\frac{1}{3}$

e) $2\frac{3}{4}$ **f)** $2\frac{2}{3}$ **g)** $4\frac{2}{5}$ **h)** $6\frac{7}{9}$

i) 1 **j)** $4\frac{1}{10}$ **k)** 0 **l)** $5\frac{1}{3}$

2. Convert the following mixed numbers into improper fractions:

a) $\frac{31}{4}$ **b)** $\frac{11}{2}$ **c)** $\frac{8}{7}$ **d)** $\frac{47}{9}$

e) $\frac{34}{5}$ **f)** $\frac{28}{3}$ **g)** $\frac{20}{7}$ **h)** $\frac{10}{7}$

i) $\frac{3}{1}$ **j)** $\frac{37}{5}$ **k)** $\frac{78}{9}$ **l)** $\frac{84}{8}$

3. Convert the following mixed numbers into improper fractions and vice versa: (Hint: Simplify before beginning the question.)

a) $27\frac{7}{9}$ **b)** $1\frac{11}{24}$ **c)** $\frac{62}{13}$ **d)** $\frac{17}{5}$

e) $\frac{19}{12}$ **f)** $2\frac{1}{2}$ **g)** $\frac{11}{4}$ **h)** $2\frac{18}{19}$

4. Are the following fractions equal? Justify your answers using diagrams, words, or numbers.

a) yes **b)** **yes** **c)** **yes** **d)** **yes**

5. Explain using words and pictures, why you can never divide by 0?

   Suppose we had 6 pieces of candy and we wanted to split them equally between 0 people.
   Since we can't divide a set of objects among 0 people, we can't divide by 0.

6. Simplify the following fraction, $3\frac{6}{3}$ = 5.