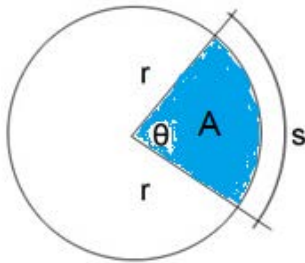# Area and Arc Length of a Sector
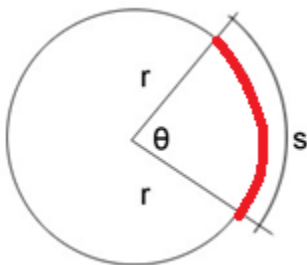
Calculating the Area of a Sector:

Area, A, of a sector, with radius. r, and subtended angle, θ, in radians is given by:

$$A = \frac{1}{2}\theta r^2$$

Note: if θ is given in degrees, it must be converted into radians first. The following formula can be used:

$$A = \frac{1}{2}\theta^0 \left(\frac{\pi}{180^0}\right)r^2$$

Calculating the Arc Length of a Sector:

Arc length, s, of a sector with radius, r, and subtended angle, θ, in radians is given by:
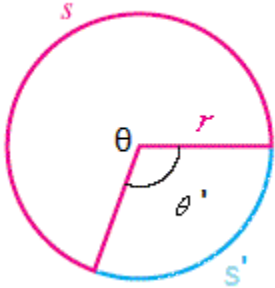
$$S = \theta r$$

Note: if θ is given in degrees, it must be converted into radians first. The following formula can be used:

$$S = \theta^0 \left(\frac{\pi}{180^0}\right)r$$

Example 3.4.

Find the lengths of the arcs s and s' in the figure if r = 4 and θ'= 60°.

Solution:

To find the arc length, s, first we have to find the angle θ that subtends the arc s,

$\theta + \theta' = 360^0$

$\theta = 360^0 - \theta' = 360^0 - 60^0 = 300^0$

Now, we can apply the formula for finding the length of an arc if the angle is given in degrees.

To find the length of s:

$s = \theta^0(\frac{\pi}{180^0})r$

$= 300(\frac{\pi}{180^0})(4)$

$= \frac{20\pi}{3}$

To find the length of s':

$s' = \theta'^0(\frac{\pi}{180^0})(r)$

$= 60(\frac{\pi}{180^0})(4)$

$= \frac{4\pi}{3}$

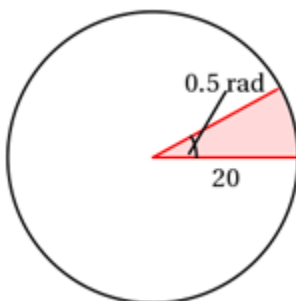**Practice Questions:**

1. Find the radius r of the circle in the figure with arc length s.

    $s = 12$

    3 rad

    r
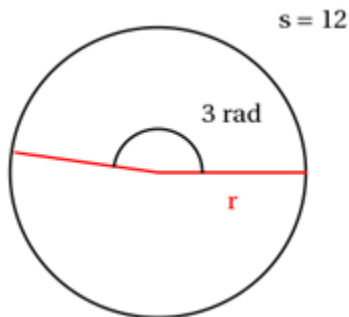
2. Find the length of an arc that subtends a central angle of 3 rad in a circle of radius 8 mi.

3. Find the area of the sectors in the following diagrams:

    $77°$

    5

    a)

    0.5 rad

    20
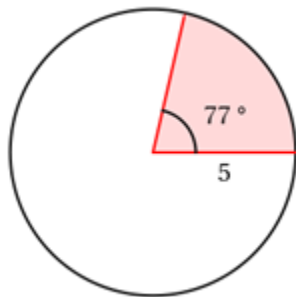
    b)

4. Find the area of a sector with central angle 1 rad in a circle of radius 14 m.

5. The area of a sector of a circle with a central angle of 4 rad is 8 m^2. Find the radius of the circle.

Area and Arc Length of a Sector

6. The Greek mathematician Eratosthenes (ca. 276-195 B.C.) measured the circumference of the earth from the following observations. He noticed that on a certain day the sun shone directly down a deep well in Syene (modern Aswan). At the same time in Alexandria, 500 miles north (on the same meridian), the rays of the sun shone at an angle of 7.2° to the zenith.
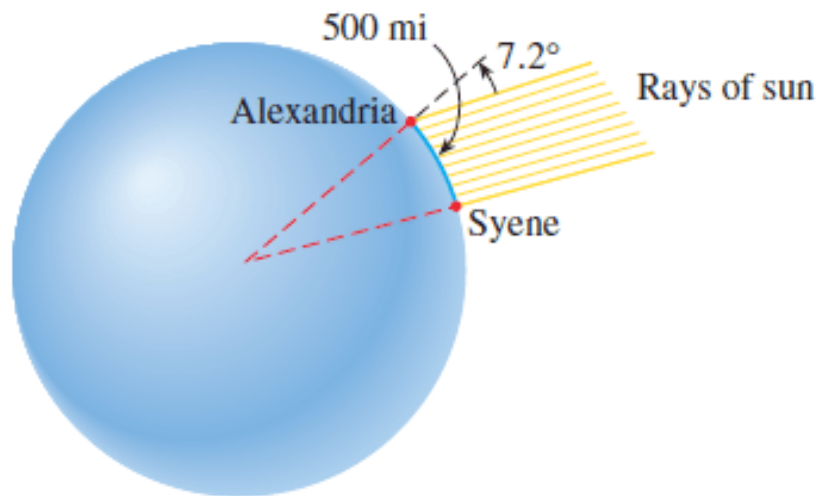
a) Use this information and the figure above to find the radius of the earth. (Round your answer to the nearest ten miles.)
b) Find the circumference of the earth.

<u>Answers:</u>

1) 4
2) 24 mi
3) a) 5.35π
   b) 100
4) 98 m²
5) 2 m
6. a) 3978.9 mi
b) 25000.2 mi

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