Operations with Algebraic Expressions: Multiplication of Polynomials

<u>The product of a monomial x monomial</u>

> To multiply a monomial times a monomial, **multiply the coefficients** and **add the exponents on powers with the same variable as a base**.

Note: A common mistake that many students make is to multiply the exponents on powers with the same variables as a base. This is NOT CORRECT. Remember the exponent rules!

Exponent Rules			
Case	What to do	Rule	Example
Multiplying powers with the same base	Add the exponents. Keep the base the same.	$(x^a)(x^b) = x^{a+b}$	$(2^5)(2^3) = 2^8$
Dividing powers with the same base	Subtract the exponents. Keep the base the same.	$\frac{x^a}{x^b} = x^a \div x^b = x^{a-b}$	$\frac{2^5}{2^3} = 2^5 \div 2^3 = 2^2$
Simplifying power of a power	Multiply the exponents. Keep the base the same.	$(x^a)^b = x^{ab}$	$(2^5)^3 = 2^{15}$
Exponent of 0	Anything to the exponent of 0 equals 1.	$x^0 = 1$	$2^0 = 1$

Other cases that come up when working with powers		
Case	What to do	Example
Adding powers with the same base and SAME exponents	The powers are like terms. Add the coefficients; keep the base and the exponent the same.	$x^a + x^a = 2x^a$
Adding powers with the same base and DIFFERENT exponents	The powers are NOT like terms. They can NOT be added.	$x^a + x^b = x^a + x^b$
Subtracting powers with the same base and SAME exponents	The powers are like terms. Subtract the coefficients; keep the base and the exponent the same.	$2x^a - x^a = x^a$
Subtracting powers with the same base and DIFFERENT exponents	The powers are NOT like terms. They can NOT be subtracted.	$x^a - x^b = x^a - x^b$

Multiplication of Polynomials

Example 1:

Simplify. $5x^3\,(-6x)$

$= 5(-6)x^{3+1}$ Multiply the coefficients. Add the exponents since both powers have base x.

$= -30x^4$

Example 2:

Simplify. $20a^2y^{-3}b\,(\frac{1}{4}ay^4)$

$= 20(\frac{1}{4})a^{2+1}y^{-3+4}b$ Multiply the coefficients. Add the exponents for powers with base a. Add the exponents for powers with base y.

$= 5a^3yb$

Example 3:

Simplify. $0.10p^{10}q^4\,(10)p^5q^{-4}$

$= 0.10(10)p^{10+5}q^{4-4}$ Multiply the coefficients. Add the exponents for powers with base a. Add the exponents for powers with base y.

$= 1p^{15}q^0$ Simplify q^0. Any number to the exponent of 0 equals 1.

$= p^{15}(1)$ p^{15} multiplied by 1 is just p^{15}

$= p^{15}$

<u>The product of a monomial x binomial</u>

To multiply a monomial by a binomial, **multiply the monomial by EVERY term making up the binomial.**

Remember: To find the product of two terms, multiply the coefficients and **add the exponents on powers with the same variable as a base**.

Example 1:

Expand. $5x^3(7x^2 + 15xy)$

$5x^3(7x^2 + 15xy)$ Step 1: Multiply the monomial by EVERY term making up the binomial.

$= 5x^3(7x^2) + 5x^3(15xy)$ Step 2: To find the product of two terms, multiply the coefficients and add the exponents on powers with the same variable as a base.

$= 35x^5 + 75x^4y$

Example 2:

Expand. $-2c^4p(10c^3p^2 - 4c)$

$-2c^4p(10c^3p^2 - 4c)$

Step 1: Multiply the monomial by EVERY term making up the binomial.

$= -2c^4p(10c^3p^2) + (-2c^4p)(-4c)$

Step 2: To find the product of two terms, multiply the coefficients and add the exponents on powers with the same variable as a base.

$= -20c^7p^3 + 8c^5p$

The product of a monomial x trinomial OR monomial x polynomial

To **multiply a monomial by a trinomial or any polynomial, multiply EVERY term in the trinomial or polynomial by the monomial**.

To find the product of two terms, multiply the coefficients and add the exponents on powers with the same variable as a base.

Example:

Expand. $7x^3(19x^7y + 20 - 3x + y)$

$7x^3(19x^7y + 20 - 3x + y)$

Step 1: Multiply the monomial by EVERY term making up the binomial.

$= 7x^3(19x^7y) + 7x^3(20) + 7x^3(-3x) + 7x^3(y)$

Step 2: To find the product of two terms, multiply the coefficients and add the exponents on powers with the same variable as a base.

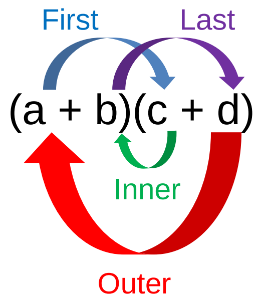
$= 133x^{10}y + 140x^3 - 21x^4 + 7x^3y$

The product of a binomial x binomial

To multiply a binomial by a binomial, multiply EVERY term in the first binomial by EVERY term in the second binomial. Then simplify by collecting (adding or subtracting) like terms, if it is possible.

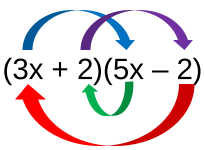
Multiplication of Polynomials

You can use the **FOIL** (First, Outer, Inner, Last) **method** to remember how to multiply binomials.



(a + b)(c + d) = ac + ad + bc + bd

Example 1:
Expand. (3x + 2)(5x – 2)



(3x + 2)(5x – 2)

Step 1: Use FOIL to multiply every term in the first binomial by every term in the second binomial.

= 3x(5x) + (3x)(−2) + (2)(5x) + (2)(−2)

Step 2: Evaluate every product.

= $15x^2$ + (−6x) + 10x + (−4)

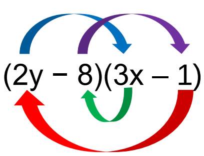
= $15x^2$ −6x + 10x −4

Step 3: Collect like terms.

= $15x^2$ + **4x** −4

This is the final answer.

Example 2:
Expand. (2y − 8)(3x – 1)



(2y − 8)(3x – 1)

Step 1: Use FOIL to multiply every term in the first binomial by every term in the second binomial.

= 2y(3x) + (2y)(−1) + (−8)(3x)+ (−8)(−1)

Step 2: Evaluate every product.

= 6yx + (– 2y) + (–24x) + (8)

There are no like terms that can be collected. Simplify double signs. Arrange terms in alphabetical order*.

= 6yx – 24x – 2y + 8

This is the final answer.

*Note: By convention, terms are written from *highest* to *lowest degree* and in alphabetical order..

Multiplication of Polynomials

Squaring a binomial

To square a binomial means to multiply the binomial by itself.

The rules of multiplying a binomial by a binomial apply. To multiply a binomial by a binomial, multiply EVERY term in the first binomial by EVERY term in the second binomial.

When multiplying a binomial by itself, the expanding follows a pattern as shown below.

$$\mathbf{(a+b)^2 =} (a+b)(a+b) = a^2 + ab + ab + b^2 = \mathbf{a^2 + 2ab + b^2}$$

Example 1:

Expand. $(5x - 3)^2$

Solution 1:

$(5x-3)^2 = (5x-3)(5x-3)$ — Step 1: Use FOIL method to expand.

$= 25x^2 - 15x - 15x + 9$ — Step 2: Collect like terms.

$= 25x^2 - 30x + 9$

Solution 2:

$(5x - 3)^2 = (5x)^2 + 2(5x)(-3) + (-3)^2$ — Step 1: Use the $(a + b)^2 = a^2 + 2ab + b^2$ pattern to expand.

$(a + b)^2 = a^2 + 2ab + b^2$

$= 25x^2 – 30x + 9$ — Step 2: Simplify each term.

Example 2:

Expand. $(9y + 2)^2$

$(9y + 2)^2 = (9y)^2 + 2(9y)(2) + 2^2$ — Step 1: Use the $(a + b)^2 = a^2 + 2ab + b^2$ pattern to expand

$= 81y^2 + 36y + 4$ — Step 2: Simplify each term.

Multiplication of Polynomials

Practice Questions:

1. Simplify each of the following algebraic expressions:

a) $3b^2a(-2b^3)$

b) $-p^4r(-21pr^2)$

c) $22a(b^2a - 2b^3a)$

d) $19x(-y^2 + 3x^3z)$

e) $3s(2s + 4qs^2 - 8)$

f) $-2(11x^2 + 20xy - 14y^3)$

g) $-(28q^3s^9 + 2q^2s - 10q^5s^3 + 18q + 9)$

h) $(2x + 3)(19x - 1)$

i) $(3x + 5)(7 - 3x)$

j) $(9y^2 + 8)(3y - 2)$

k) $(5y - 3)(y^3 + 6)$

l) $(8a - 3)(9a - 10)$

m) $(9x + 2)^2$

n) $(14 - y)^2$

o) $(x^2 - y)^2$

p) $(4a^3 - 3b)^2$

Answers:

1. a) $-6b^5a$

b) $21p^5r^3$

c) $22b^2a^2 - 44b^3a^2$

d) $57x^4z - 19xy^2$

e) $6s^2 + 12qs^3 - 24s$

f) $-22x^2 - 40xy + 28y^3$

g) $-28q^3s^9 - 2q^2s + 10q^5s^3 - 18q - 9$

h) $38x^2 + 55x - 3$

i) $-9x^2 + 6x + 35$

j) $27y^3 - 18y^2 + 24y - 16$

k) $5y^4 - 3y^3 + 30y - 18$

l) $72a^2 - 107a - 30$

m) $81x^2 + 36x + 4$

n) $y^2 - 28y + 196$

o) $x^4 - 2x^2y + y^2$

p) $36a^6 - 24a^3b + 9b^2$