Common Factoring

Introduction to Factoring

Factoring is the OPPOSITE operation of expanding algebraic expressions.

To factor an algebraic expression means to find the numbers, monomials, binomials or polynomials that multiplied together result in the given algebraic expression.

Common Factoring

Step 1: Find the **greatest common factor of all terms** in the algebraic expression. Consider the numbers and variables making up each term.

Step 2: Write the common factor in front of the brackets. In brackets, write the algebraic expression resulting from dividing EACH term by the common factor.

Example 1:

Factor $16xy^2 + 20x^2y - 4x^3y^2$.

Step 1: Find the greatest common factor of $16xy^2$, $20x^2y$ and $4x^3y^2$.

Look at the numbers 16, 20 and 4. The greatest common factor of these numbers is 4, since all of these numbers can be divided by 4 evenly.

$16 \div 4 = 4$

$20 \div 4 = 5$

$4 \div 4 = 1$

Look at the variables in each term, xy^2, x^2y and x^3y^2. The greatest common factor of these variables is xy since all of these terms can be divided by xy evenly.

$xy^2 \div xy = y$

$x^2y \div xy = x$

$x^3y^2 \div xy = x^2y$

(Note: the terms are divided according to exponent rules).

Therefore, the greatest common factor is 4xy.

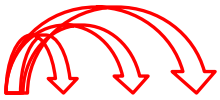
Step 2: Write the greatest common factor in front of brackets. Determine the algebraic expression in brackets by dividing each term in the given algebraic expression by the greatest common factor.

$$4xy\left(\frac{16xy^2}{4xy} + \frac{20x^2y}{4xy} - \frac{4x^3y^2}{4xy}\right)$$

$= 4xy\,(4y + 5x - x^2y)$

Common Factoring

Step 3 (Optional): Double check the answer by expanding $4xy(4y + 5x – x^2y)$. The answer should be the algebraic expression that was given in the question.



$4xy(4y + 5x – x^2y)$

$= 4xy(4y) + 4xy(5x) + 4xy(–x^2y)$

$= 16xy^2 + 20x^2y – 4x^3y^2$

Example 2:

Factor $–144p^5q – 12p^3 – 6p$.

Step 1: Find the greatest common factor of $–144p^5q$, $–12p^3$ and $–6p$.

The greatest common factor of –144, –12 and −6 is −6.

$–144 \div (–6) = 24$

$–12 \div (–6) = 2$

$–6 \div (–6) = 1$

The greatest common factor of p^5q, p^3 and p is p.

$p^5q \div p = p^4q$

$p^3 \div p = p^2$

$p \div p = 1$

Therefore, the greatest common factor is −6p.

Step 2: Write the greatest common factor in front of brackets. Determine the algebraic expression in brackets by dividing each term in the given algebraic expression by the greatest common factor.

$$–6p\left(\frac{–144p^5q}{–6p} – \frac{12p^3}{–6p} – \frac{6p}{–6p}\right)$$

$= –6p (24p^4q + 2p^2 + 1)$

Practice questions:

1. Factor the following algebraic expressions. Check your work by expanding.

a) $24k^3s^2 + 12k^2s^5 – 48k^2s^3$
b) $10d^4c^3 + 50d^3c^4 – 40d^4c^2 + 5d^3c^2$
c) $12x^3 + 6x – 2y$
d) $81c^4 + c^2 + 10d$
e) $36q^3r + 90rq^2 + 81rq$
f) $3x^3 – 6x^2 + 20y – 4y^2$

Answers:

1. a) $12k^2s^2 (2k + s^3 –4s)$
b) $5d^3c^2(2dc + 10c^2 –8d +1)$
c) $2(6x^3 + 3x –y)$
d) $c^2 (81c^2 + 1) + 10d$
e) $9qr(4q^2 +10q +9)$
f) $3x^2 (x – 2) + 4y(5–y)$