ACCUPLACER Quantitative Reasoning, Algebra, and Statistics (QAS) Placement Test Study Guide

George Brown College Assessment Centre

GEORGE BROWN COLLEGE Assessment Centre

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Please note that the **guide is for reference only and that it does not represent an exact match with the assessment content**. The Assessment Centre at George Brown College is not responsible for students' assessment results.

INTRODUCTION

The ACCUPLACER Quantitative Reasoning, Algebra, and Statistics Placement Test Study Guide is a reference tool for George Brown College students in getting ready for the test.

The study guide does not cover all topics on the assessment.

The ACCUPLACER Quantitative Reasoning, Algebra, and Statistics Placement Test Study Guide is based on the curriculum provided by ACCUPLACER.

It is recommended that users follow the guide in a step-by-step order as one topic builds on the previous one. Each section contains theory, examples with full solutions and practice questions. Answers to practice questions can be found in Appendix B.

Reading comprehension and understanding of terminology are an important part of learning mathematics. In the text, words in **bold** represent terms that are important to know and are included in the glossary of mathematical terms in Appendix A.

The Assessment

The ACCUPLACER Quantitative Reasoning, Algebra, and Statistics Placement Test is:

- Computer based with 20 multiple choice questions (you must book a Placement Test with the Assessment Centre)
- Computer adaptive, which means the level (and corresponding marks) of each subsequent question is dependent on how well the current question is answered

Note: Handheld calculators are **not** allowed on the assessment. For those questions where a calculator is allowed, a calculator icon will appear in the top right of the screen.

In this study guide, the phrase “calculator allowed” accompanying a question indicates where you might have access to a calculator.

Additional Resources

ACCUPLACER (the company that creates the test) has an online practice app that provides typical questions asked in a multiple choice format with full solutions. Be sure to select the ‘Next Generation’ and work on sample questions for the Quantitative, Reasoning and Statistics (QAS) test.

This study guide provides introductory material for most topics that the Assessment is based on. Further study is recommended.

1. RATIONAL NUMBERS

Definitions

A **rational number** is a number that can be written as a **ratio** (or **fraction**).
It can be a positive or negative quantity.
A positive or negative whole number is called an **integer**.

For example, the following numbers are all rational numbers:

$$3, -2, \frac{1}{4}, 0.056, -1\frac{2}{3}, -87.9$$

Note: a negative quantity is indicated with a minus sign '-' before the number, whereas either a plus '+' or no sign indicates a positive quantity. Using brackets around a rational number emphasizes the sign of the number. Confusion can arise sometimes whether a minus '-' sign represents a negative number (a quantity) or a subtraction (an operation).

Also, please note that a calculator may not be provided for questions in this topic.

Operations on Rational Numbers

Definitions

Sum: the answer to an addition (also: total, add, increase by)

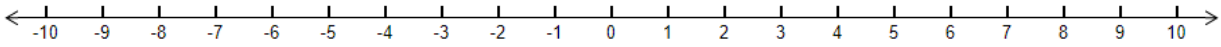
Difference: the answer to a subtraction (also: take away, subtract, decrease by)

Product: the answer to a multiplication (also: times, multiply, of)

Quotient: the answer to a division (also: share, divide)

Adding or Subtracting Rational Numbers

Rational numbers can be represented on a number line:



When we are adding or subtracting rational numbers, we move along the number line, either to the right or to the left.

When adding or subtracting rational numbers, the first value represents the starting point on the number line and the combination of the operation (adding or subtracting) and the second value (positive or negative) indicates which direction to move along the number line (either left or right).

Adding a positive number is addition and has us move towards the right on the number line.

Adding a negative number is a definition of subtraction and the combination has us move to the left on the number line.

Subtracting a positive number also defines subtraction and again has us move to the left on the number line.

Subtracting a negative number results in addition (the two negatives cancel to make a positive) and so has us move to the right on the number line.

Examples

Example 1:

$$(+2) - (-5) + (-7) - (+4) =$$

Solution 1:

Brackets (or parenthesis) are helpful in identifying the sign of a number. In solving these types of expressions, our first step might be to combine the operation (adding or subtracting) with the sign of the number (positive or negative) using the rules listed above:

$$(+2) - (-5) + (-7) - (+4) = 2 + 5 - 7 - 4 = -4$$

Example 2:

$$\left(+\frac{1}{2}\right) - \left(-\frac{2}{3}\right) =$$

Solution 2:

If fractions are involved, recall that you must have **common denominators** before you can add or subtract the **numerators**. The rules for adding or subtracting positive or negative numbers is the same as above.

$$\left(+\frac{1}{2}\right)-\left(-\frac{2}{3}\right)=\left(+\frac{1}{2}\right)+\frac{2}{3}$$
$$=\frac{3}{6}+\frac{4}{6}$$
$$=\frac{7}{6}$$
$$=1\frac{1}{6}$$

Note: It is important that answers to questions are given in their simplest, or reduced form. If the answer to a fraction operation is $\frac{6}{8}$, its reduced form will be $\frac{3}{4}$. Both answers could be given in a multiple-choice assessment, but $\frac{3}{4}$ will be the answer marked correctly.

Multiplying or Dividing

The rule for multiplying and dividing rational numbers is: “Every pair of like signs produces a positive”. This means that if there is an odd number of negatives, the final answer will be negative.

Note: We can represent multiplication in several ways. We can use a times sign or brackets next to each other or with a centre dot. For example, 3 times 2 can be written as:

3×2 or $(3)(2)$ or $3 \cdot 2$

Division can be represented using a division sign or with either a horizontal line or a forward slash. For example, 3 divided by 2 can be written as:

$3 \div 2$ or $\frac{3}{2}$ or $3/2$

Examples

Example 3:

$$\frac{(+3)\times(-2)\times(-4)}{(8)\div(-2)}=$$

Solution 3:

With a rational (or fraction) expression, simplify the numerator and denominator separately first, then combine them as a quotient,

$$(+3) \times (-2) \times (-4) = 24$$

$$(8) \div (-2) = -4$$

Then,

$$\frac{(+3)\times(-2)\times(-4)}{(8)\div(-2)} = \frac{24}{-4} = -6$$

Example 4:

$$\left(-\frac{3}{8}\right) \times \left(+\frac{2}{7}\right) =$$

Solution 4:

If fractions are being multiplied, recall that you multiply the numerators and multiply the denominators. Always <u>reduce</u> the fraction to its simplest terms, if possible.

$$\left(-\frac{3}{8}\right) \times \left(+\frac{2}{7}\right) = \frac{-3\times2}{8\times7} = \frac{-6}{56} = \frac{-3}{28}$$

Example 5:

$$\left(-2\frac{1}{8}\right) \div \left(-5\frac{5}{6}\right) =$$

Solution 5:

For division by fractions, you multiply the first quantity by the **reciprocal** ('flip') of the second quantity,

$$\left(-2\frac{1}{8}\right) \div \left(-5\frac{5}{6}\right) = \frac{-17}{8} \div \frac{-35}{6} = \frac{-17}{8} \times \frac{6}{-35} = \frac{-102}{-280} = \frac{51}{140}$$

Note: for multiplying and dividing **mixed numbers/fractions**, first convert them to **improper fractions**.

Exponents

Definitions

An **exponent** is a way of showing repeated multiplication.

For example, we can write $2 \times 2 \times 2$ as 2^3. We say this as "2 to the exponent (or power) 3". The number being repeatedly multiplied is called the **base**. The exponent is written as a smaller number above and to the right of the base.

Examples

Example 6:

Evaluate $(-2)^3$

Solution 6:

$$(-2)^3 = (-2) \times (-2) \times (-2) = -8$$

Example 7:

Evaluate $(-3)^4$

Solution 7:

$$(-3)^4 = (-3) \times (-3) \times (-3) \times (-3) = 81$$

Example 8:

Evaluate -2^4

Solution 8:

$$-2^4 = -(2 \times 2 \times 2 \times 2) = -(16) = -16$$

Example 9:

Evaluate 10^5

Solution 9:

$$10^5 = 10 \times 10 \times 10 \times 10 \times 10 = 100{,}000$$

Example 10:

Evaluate $\left(\frac{1}{2}\right)^4$

Solution 10:

$$\left(\frac{1}{2}\right)^4 = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{16}$$

Order of Operations

Definitions

When there are a number of mathematical operations ($+$, $-$, $\times$, $\div$, or exponents) occurring in an expression, the operations need to be carried out in a certain order:

- If there are brackets or groupings of terms, apply order of operations here first
- If there are any exponents, apply them next
- Then apply multiplications or divisions, as they occur from left to right
- Finally, work on additions or subtractions, again as they occur from left to right

Order of operations applies in **all** math.

Note: For **rational expressions**, the numerator and denominator are evaluated separately according to order of operations, then determine the **quotient**.

Examples

Example 11:

Which expression is equivalent to $-2(3-6)^2 - 2^3$?

A. 28
B. -2
C. -26
D. -10

Solution 11:

In this question, the word **equivalent** is asking you to determine which answer represents the question, but in a different form.

Following order of operations for $-2(3-6)^2 - 2^3$, we see there are brackets around the $(3-6)$ so we determine that first,

$$-2(3-6)^2 - 2^3 = -2(-3)^2 - 2^3$$

We'll then proceed to apply exponents,

$$-2(-3)^2 - 2^3 = -2(9) - 8$$

Then we apply any multiplications or divisions,

$$-2(9) - 8 = -18 - 8$$

Then finally, any additions or subtractions,

$$-18 - 8 = -26$$

So, answer C.

Example 12:

$18 - 0.5 \times 12 - 0.75 =$

Solution 12:

Applying order of operations, we see there are no brackets and no exponents. We then move onto any multiplications or divisions. There is one multiplication to do, 0.5×10,

$$18 - 0.5 \times 12 + 0.75 = 18 - 6 + 0.75$$

Now apply additions and subtractions as they occur from left to right,

$$18 - 6 + 0.75 = 12 + 0.75 = 12.75$$

Example 13:

$\frac{(5-2\times3)^2}{6-8} =$

Solution 13:

Evaluate the numerator and denominator separately, then calculate the quotient,

$$(5 - 2 \times 3)^2 = (5 - 6)^2 = (-1)^2 = 1$$

$$6 - 8 = -2$$

Then,

$$\frac{(5-2\times3)^2}{6-8} = \frac{1}{-2}$$

Note: $\frac{1}{-2}$ can be written as $\frac{-1}{2}$ or $-\frac{1}{2}$ (either of the last two representations are preferred).

Apply Rational Numbers

Definitions

Rational numbers occur in many aspects in our lives such as the weather (temperature), business (stock prices going up or down), sports (an athletes' performance), etc.

Examples

Example 14:

During the winter, the temperature in a city was $-9°\text{C}$ at night and $7°\text{C}$ during the day. What is the difference between these two temperatures?

Solution 14:

To find the difference between two quantities, we subtract. In this example, we would start with the higher temperature, $7°\text{C}$, then subtract $-9°\text{C}$,

$$7°\text{C} - (-9°\text{C}) = 7°\text{C} + 9°\text{C} = 16°\text{C}$$

Example 15:

Mauna Kea is a mountain in Hawaii, U.S. The base of the mountain starts below sea level at a depth of $-19{,}700$ feet and rises above sea level to a height of $13{,}796$ feet. What is the total height of the mountain?

Solution 15:

Again, start with the greater height $13{,}796$ feet, and subtract the lower $-19{,}700$ feet,

$$13{,}796 \text{ feet} - (-19{,}700) \text{ feet} = 13{,}796 + 19{,}700 \text{ feet} = 33{,}496 \text{ feet}$$

Absolute Value

Definitions

The **absolute value** of a number (or quantity) is the distance that number (or quantity) is from zero. We are only interested in distance, not direction, so it does not matter which side of zero the number (or quantity) is.

The absolute value of 3 is 3. The absolute value of -3 is also 3.

We use vertical bars on either side of the number (or quantity) to represent absolute value. The absolute value of 3 would be written as $|3|$. The absolute value of -3 would be written as $|-3|$.

Examples

Example 16:

$$|4| =$$

Solution 16:

In words, the absolute value of 4 represents the number of units 4 is from zero, which is 4.

$$|4| = 4$$

Example 17:

$$|-5| =$$

Solution 17:

In words, the absolute value of -5 represents the number of units -5 is from zero, which is 5,

$$|-5| = 5$$

Example 18:

$$|7 - 3| =$$

Solution 18:

Absolute value bars work in a similar way to brackets, you follow the order of operations in the absolute value bars first, then determine the absolute value of the answer,

$$|7 - 3| = |4| = 4$$

Example 19:

$$-2|3 - 8|=$$

Solution 19:

Again, evaluate inside the absolute value bars first, then determine the absolute value, then apply the multiplication of -2,

$$-2|3 - 8| = -2|-5| = -2(5) = -10$$

Example 20:

Which expression is equivalent to $|-15+8|$?

A. $|-15|+|8|$
B. $-|15|-|-8|$
C. $|15|-|8|$
D. $|-15|-|8|$

Solution 20:

For this question, it might be best to evaluate the question, then compare each of the possible answers to see which match.

$|-15+8| = |-7| = 7$

Now see what each of the possible answers give:

A. $|-15|+|8| = (15)+(8) = 23$
B. $-|15|-|-8| = -(15)-(8) = -23$
C. $|15|-|8| = (15)-(8) = 7$
D. $-|-15|-|8| = -(15)-(8) = -23$

So, C. gives the equivalent answer.

2. RATIO AND PROPORTIONAL RELATIONSHIPS

Calculate with Ratios

Definitions

A ratio is a comparison of two or more quantities that have the same measurement units. Those units are not typically included as part of the final ratio. A ratio can be written using a colon (:) to separate the numbers, or as a fraction if there are only two quantities.

The order of a ratio is determined by the information in the problem. For instance, if the ratio of the length to the width of a room is given as 5:2, the 5 corresponds to the length and the 2 to the width.

As with fractions, ratios are usually reduced to their simplest form. For instance, the ratio 2:10:16 would be simplified to 1:5:8.

Examples

Example 21:

Write the following comparison as a ratio in its simplest, or reduced, form: 35 cents to 50 cents.

Solution 21:

Most often, we write the ratio as a fraction to allow it to be reduced more easily,

$$35 \text{ cents to } 50 \text{ cents} = 35:50 = \frac{35}{50} = \frac{7}{10}$$

Note: When comparing two quantities that have the same units, the units cancel on the top and bottom of the fraction and so do not appear in the answer.

Calculate with Rates

Definitions

A **rate** is a comparison of two quantities that have different measurement units.

A rate is often written as a **fraction** with the units included.

A **unit rate** (or cost) describes how many units of the first type of quantity corresponds to one unit of the second type. For instance, a car drives 120 kilometres in 2 hours. This represents a rate. The unit rate is given when the second quantity is 1. So, 120 kilometres in 2 hours will become 60 kilometres in 1 hour.

Examples

Example 23:

Write as a unit rate: $315.70 earned in 22 hours. (calculator allowed)

Solution 23:

Just as with ratios, a rate can be written as a fraction, then reduce to a unit rate by calculating the quotient,

$$\frac{\$315.70}{22\text{ hours}} = \$14.35/\text{hour}$$

Note: Unit rates are often written on one line with the first quantity and its unit followed by a forward slash, then the second quantities unit.

Example 24:

If a car is driving at 30 km/h, how long will it take to drive 70 km?

Solution 24:

30 km/h is a unit rate. It indicates that the car will drive 30 kilometres in 1 hour. To determine how long it will take to drive 70 kilometres, we need to find how many times 30 divides into 70,

$$\frac{70\text{km}}{30\text{km/h}} = \frac{70\text{ kilometres}}{1} \times \frac{1\text{ hour}}{30\text{ kilometres}} = \frac{70}{30}\text{hours} = \frac{7}{3}\text{hours} = 2\frac{1}{3}\text{hours}$$

Calculating with Proportions

Definitions

A proportion is the equality of two rates or two ratios.

For example, $50 in two hours is equivalent to $25 in one hour. Or a ratio of 14 business students to 10 non-business students in a class is equivalent to 7 business students to 5 non-business students in another class.

There is a total of four quantities in a proportion. The numerator and denominator of one fraction, plus the numerator and denominator of the other. When one of those quantities is missing and is to be determined, we then solve the proportion. Solving a proportion is an important skill in math. The following method illustrates using the **cross product.**

The cross product method is used when one fraction is equal to another fraction as in a proportion. The cross product is when the numerator of the left-hand fraction multiplies the denominator of the right-hand fraction and, at the same time, we multiply the numerator of the right hand fraction by the denominator of the left hand fraction.

Examples

Example 25:

Solve the proportion $\frac{n}{20} = \frac{5}{6}$

Solution 25:

We start by determining the cross product, then look to isolate the unknown, *n*,

Step:	In words:	The equation will look like:
1	Start with the equation	$\frac{n}{20} = \frac{5}{6}$
2	Write out the cross product	$n \times 6 = 5 \times 20$
3	Simplify	$6n = 100$
4	Divide both sides by the number multiplying *n*	$\frac{6n}{6} = \frac{100}{6}$
5	Simplify for the answer	$n = \frac{50}{3}$ or $16\frac{2}{3}$

Example 27:

The ratio of the length of a room to its width is 5:2. If the length of the room is 3.2 metres, what will the width be?

Solution 27:

We can set these equivalent ratios as a proportion and let *n* be the width of the room,

Step:	In words:	The equation will look like:
1	Start with the equation	$\frac{5}{2} = \frac{3.2}{n}$
2	Write out the cross product	$5 \times n = 3.2 \times 2$
3	Simplify	$5n = 6.4$
4	Divide both sides by the number multiplying *n*	$\frac{5n}{5} = \frac{6.4}{5}$
5	Simplify for the answer	$n = 1.28$

So, the answer is that the width will be 1.28 metres.

Unit Conversions

Definitions

Converting from one unit of measurement to another requires knowing the **conversion rate** which is usually provided in the question.

One method for converting units is to use the **factor-label method** illustrated below. We set up conversion rates so that we can cancel the units we do not want. So, pay very close attention to the units.

A unit can be canceled out in the multiplication if the unit appears in the numerator somewhere and in the denominator somewhere.

Examples

Example 28:

Convert 27 feet to yards. (1 yard = 3 feet)

Solution 28:

Using the factor-label method,

Step:	In words:	The equation will look like:
1	The starting units are feet (ft) and the target unit is in yards (yd)	$27\text{ ft} \rightarrow \text{ yd}$
2	We are given an equivalency of 1 yd = 3 ft. The equivalency should be written as $\frac{1\text{ yd}}{3\text{ ft}}$ so that the feet units are in the denominator and will cancel.	$27\text{ ft} \times \frac{1\text{ yd}}{3\text{ ft}}$
3	Multiply by the conversion rate. The starting feet units cancel, and the target units, yard remain.	$\frac{27}{1}\cancel{\text{ft}} \times \frac{1\text{ yd}}{3\cancel{\text{ft}}} = 9\text{ yd}$

Example 29:

What is the number of millilitres in 5 litres? (1 litre = 1000 millilitres)

Solution 29:

Using the factor-label method,

Step:	In words:	The equation will look like:
1	The starting units are litres (L) and the target unit is in millilitres (mL)	$5\text{ L} \rightarrow \text{ mL}$
2	We are given an equivalency of 1 litre = 1,000 millilitres. The equivalency should be written as $\frac{1{,}000\ mL}{1\ L}$ so that the litre units are in the denominator and will cancel.	$5\text{ L} \times \frac{1{,}000\text{ mL}}{1\text{ L}}$
3	Multiply by the conversion rate. The starting units, litres, cancel, and the target units, millilitres remain.	$\frac{5}{1}\cancel{\text{L}} \times \frac{1{,}000\text{ mL}}{1\cancel{\text{L}}} = 5{,}000\text{mL}$

3. EXPONENTS

Definitions

An **exponent** was defined earlier in section 1. Rational Numbers. An exponent is a way of showing repeated multiplication.

In this section we will see that exponents can also be a fraction and that the base can be a **variable**.

Exponents that are fractions are a way of representing a **radical expression** (see later in this section).

Note: If there is no exponent written, it is assumed to be 1. For example, x appears to have no exponent, but it can be re-written as x^1 without changing the value.

Calculate with Exponents

To calculate or simplify expressions with exponents, it is helpful to use one or more of the following exponent rules.

Exponent rules

Product Rule

Recall that in math, the product is the answer to a multiplication.

Example 31:

$$x^3 \cdot x^2 =$$

Solution 31:

One way to see how this expression simplifies is to write it in expanded form:

$$x^3 \cdot x^2 = (x \cdot x \cdot x) \cdot (x \cdot x) = x^5$$

By using expanded form, we can see that a shortcut would be to add the exponents in the question:

$$x^3 \cdot x^2 = x^{3+2} = x^5$$

This rule <u>only</u> works when the bases are the same.

We can now write the product rule for exponents as:

$$a^m \times a^n = a^{m+n}$$

In words, the rule says: When the bases are the same and being multiplied, we keep the base and add the exponents.

Quotient Rule

Recall that the quotient is the answer to a division.

Example 32:

Simplify $x^8 \div x^5$

Solution 32:

Again, we could write this out in expanded form. Also, we will write the division out using a horizontal (fraction) line:

$$x^8 \div x^5 = \frac{x^8}{x^5} = \frac{x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x \cdot x \cdot x}$$

For every x in the numerator, we can cancel (divide) by an x in the denominator:

$$x^8 \div x^5 = \frac{x^8}{x^5} = \frac{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot x \cdot x \cdot x}{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot \cancel{x}}$$

We are left with 3 x's in the numerator which can be represented by x^3. We can see that a shortcut would be to subtract the exponents in the question:

$$x^8 \div x^5 = x^{8-5} = x^3$$

We can now write the quotient rule for exponents as:

$$a^m \div a^n = a^{m-n}$$

In words, the rule says: When the bases are the same and being divided, we keep the base and subtract the exponents.

Power of a power

Now we introduce brackets. We call an expression such as $(x^3)^4$ a power of a power.

Example 33:

Simplify $(x^3)^4$

Solution 33:

Writing this in expanded form we can see:

$$(x^3)^4 = (x^3)(x^3)(x^3)(x^3)$$

Using the product rule, we can add the exponents together:

$$(x^3)^4 = (x^3)(x^3)(x^3)(x^3) = x^{3+3+3+3} = x^{12}$$

The shortcut for a power to a power is to multiply the exponents together:

$$(x^3)^4 = x^{3\times 4} = x^{12}$$

We can now write the power of a power rule as:

$$(a^m) = a^{mn}$$

In words, the rule says: When a base is to the power of one exponent, and then to the power of another exponent, we keep the base and multiply the exponents.

Zero as an exponent

This rule comes as a particular outcome of the Quotient Rule. What happens if both the bases and the exponents are the same?

Example 34:

Simplify $x^5 \div x^5$

Solution 34:

Using expanded form, we can see that the answer is 1:

$$x^5 \div x^5 = \frac{x^5}{x^5} = \frac{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}}{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}} = 1$$

It is possible to do this with several examples and note that when both the bases and the exponents are the same, the expression will simplify to 1.

We can now generalize and say the zero as an exponent rule is:

$$a^0 = 1$$

In words, anything (except 0 as a base) to the power zero is 1.

Negative Exponents

Again, this rule is an outcome of using the Quotient rule. This occurs when same bases are being divided but the exponent on the numerator base is smaller than the exponent on the denominator base.

Example 35:

Simplify $x^5 \div x^8$

Solution 35:

We can apply the quotient exponent rule,

$$x^5 \div x^8 = x^{5-8} = x^{-3}$$

But what does x^{-3} mean?
If we use the expanded form, we will see that after all x's have been cancelled, there are 3 x's left in the denominator,

$$x^5 \div x^8 = \frac{x^5}{x^8} = \frac{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}}{\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot x\cdot x\cdot x} = \frac{1}{x\cdot x\cdot x} = \frac{1}{x^3}$$

We can see that x^{-3} is equivalent to $\frac{1}{x^3}$

We can write the negative exponent rule as:

$$a^{-n} = \frac{1}{a^n} \qquad \text{or} \qquad \frac{1}{a^{-n}} = a^n$$

In words, a negative exponent shifts the base and its exponent from the numerator to the denominator, or vice-versa.

Examples

Example 36:

$$10^{-3} =$$

Solution 36:

$$10^{-3} = \frac{1}{10^3} = \frac{1}{10\cdot 10\cdot 10} = \frac{1}{1000}$$

Example 37:

Simplify $(2x^2)^3$

Solution 37:

In some cases when there is an exponent on the outside of the bracket, it may be helpful to write it out in expanded form,

$(2x^2)^3 = (2x^2)(2x^2)(2x^2) = 8x^6$

An alternative method is to use the power of a power rule, making sure to distribute the outside exponent (3) on each part of the term inside the bracket,

$(2x^2)^3 = (2)^3 \cdot (x^2)^3 = 8x^6$

Example 38:

$$\frac{(3x^2y)(2x^0y^3)^2}{(2x^{-3}y^5)} =$$

Solution 38:

As with order of operations, we simplify the numerator and denominator separately first, so we'll start with simplifying the numerator $(3x^2y)(2x^0y^3)^2$. Look for any zero exponents first, as we can replace these using the rule $a^0 = 1$,

$(3x^2y)(2x^0y^3)^2 = (3x^2y)(2(1)y^3)^2 = (3x^2y)(2y^3)^2$

Next, apply the power of a power rule $(a^m) = a^{mn}$,

$(3x^2y)(2y^3)^2 = (3x^2y)(2)^2(y^3)^2 = (3x^2y)(4y^6)$

Then apply the product rule $a^m \times a^n = a^{m+n}$,

$(3x^2y)(4y^6) = 12x^2y^7$

Now we have the numerator simplified, we look at the denominator, $(2x^{-3}y^5)$.

Here we see there is no work to simplify so we now consider the quotient as,

$$\frac{12x^2y^7}{2x^{-3}y^5}$$

Where we apply the quotient rule $a^m \div a^n = a^{m-n}$ on terms that have exponents, and for the numbers at the beginning of a term, reduce them as a fraction,

$$\frac{12x^2y^7}{2x^{-3}y^5} = \left(\frac{12}{2}\right)\left(\frac{x^2}{x^{-3}}\right)\left(\frac{y^7}{y^5}\right) = 6x^5y^2$$

Calculate with Radicals

Definitions

The answer to the **square root** of a number is a number that multiplied by itself returns the original number. For example, the square root of 49 is 7 because 7×7 is 49. We can write the square root of 49 using a **radical** symbol that wraps around the number: $\sqrt{49}$

The **cube root** of a number is a number that is multiplied by itself 3 times to return the original number. For example, the cube root of 27 is 3 because $3 \times 3 \times 3$ is 27. We can also write the cube root of 27 as a radical symbol: $\sqrt[3]{27}$

A **radical** is any expression that contains a root symbol. The **radicand** is the number or variable(s) that is (are) beneath the radical sign. The number of the root, 2 for square root or 3 for cube root, is written the same size as an exponent but placed on the left and above the radical symbol.

Note: as square roots are quite common, the number 2 is usually not written in the radical symbol.

Radical Rules

Simplifying radicals

In math, we always look to simplify expressions whether they are fractions (reducing to simplest terms) or **algebraic expressions** (collecting like terms – see **4. Algebraic Expressions**). Square root radicals can also be simplified or reduced where possible. In order to do this, we need to know what a **square number** is.

A square number is the product (answer) of a number multiplied by itself. For example, if we take the number 5 and multiply it by itself, we will get the product 25. We say that 25 is a square number. The square root of a square number is always a whole number.

The following is a list of square numbers up to 100:

1, 4, 9, 16, 25, 36, 49, 64, 81, 100

Examples

Example 39:

$$\sqrt{121} =$$

Solution 39:

The square root of 121 is 11, as $11 \times 11 = 121$

Example 40:

$$\sqrt{12} =$$

Solution 40:

When the radicand is not a square number, we can still reduce it to a simpler form. Where possible, we replace the number in the radicand by a product. For instance, we could replace 12 with 1×12, 2×6 or 3×4. We choose the pair that contains the greatest square number. So we will replace 12 with 4×3. In reducing radicals its' helpful to write the square number first:

$$\sqrt{12} = \sqrt{4 \times 3}$$

Then we can separate the one radical into the product of two radicals and simplify the radical that contains the square number

$$\sqrt{12} = \sqrt{4 \times 3} = \sqrt{4} \cdot \sqrt{3} = 2\sqrt{3}$$

Adding or subtracting radicals

We can combine radicals when the radicands (the quantity inside the radical) are equal. For instance, if we have $\sqrt{7} + \sqrt{7}$ we can combine them to make $2\sqrt{7}$. Note: $\sqrt{7} + \sqrt{7}$ is not $\sqrt{14}$.

Examples

Example 44:

$$6\sqrt{20} + 7\sqrt{45} =$$

Solution 44:

We notice that the radicands are not the same so we cannot combine them directly. We look to simplify the radicals first to see if they can be made the same:

$$\sqrt{20} = \sqrt{4 \times 5} = \sqrt{4} \cdot \sqrt{5} = 2\sqrt{5}$$

$$\sqrt{45} = \sqrt{9 \times 5} = \sqrt{9} \cdot \sqrt{5} = 3\sqrt{5}$$

Now we substitute the simplified radicals back in the original question, simplify then combine:

$$6\sqrt{20} + 7\sqrt{45} = 6(2\sqrt{5}) + 7(3\sqrt{5}) = 12\sqrt{5} + 21\sqrt{5} = 33\sqrt{5}$$

Multiplying or dividing radicals

These operations can be more straightforward than addition or subtraction. The radicands do not need be the same.

Examples

Example 45:

$$\sqrt{12} \times \sqrt{3} =$$

Solution 45:

We can directly multiply the radicands together under one radical symbol, then look to see if we can simplify the radical as before:

$$\sqrt{12} \times \sqrt{3} = \sqrt{12 \times 3} = \sqrt{36} = 6$$

Example 47:

$$(3\sqrt{7})(6\sqrt{7}) =$$

Solution 47:

Separately, we can multiply the numbers in front of the radicals together, then the radicands together, then look to simplify if possible:

$$(3\sqrt{7})(6\sqrt{7}) = (3 \times 6)(\sqrt{7 \times 7}) = (18)(\sqrt{49}) = (18)(7) = 126$$

Order of operations

As with any expression or equation or function in math, the order of operation rules applies as described in **1. Rational Numbers**.

Examples

Example 48:

$$\sqrt{5}(3\sqrt{5} + \sqrt{2}) =$$

Solution 48:

$\sqrt{5}$ is multiplying all terms inside the bracket, so we distribute $\sqrt{5}$ on each of those terms,

$\sqrt{5}(3\sqrt{5}) + \sqrt{5}(\sqrt{2})$

Then we can combine the radicals together by multiplying the radicands,

$3\sqrt{5 \times 5} + \sqrt{5 \times 2} = 3\sqrt{25} + \sqrt{10}$

Where possible, simplify any radicals,

$3\sqrt{25} + \sqrt{10} = 3(5) + \sqrt{10} = 15 + \sqrt{10}$

Calculate with Fractional Exponents

Definitions

Any radical can be re-written using fractional (rational) exponents.

The denominator in the fractional exponent corresponds to the root number of the radical. The numerator in the fractional exponent corresponds to the normal exponent.

For instance, $\sqrt[3]{x^2}$ can be re-written as $x^{2/3}$.

This can be added to the other exponent rules as listed above:

$$x^{m/n} = \sqrt[n]{x^m} \qquad \text{or} \qquad x^{m/n} = \sqrt[n]{x}^m$$

Notice that the denominator, n, of the rational exponent always goes on the outside of the root sign to the left. The numerator, *m*, can either go with the radicand, or it can be placed outside the radical, above and to the right.

The exponent rules listed at the beginning of this section also apply to fractional exponents.

Examples

Example 49:

$$64^{1/3} =$$

Solution 49:

To evaluate terms with fractional exponents, it is sometimes helpful to replace the fractional exponent with a radical,

$$64^{1/3} = \sqrt[3]{64} = 4$$

Scientific Notation

Definitions

Scientific notation is a way of writing numbers or quantities, usually very large or very small quantities. This notation is used a lot in science. It allows numbers to be written in a consistent and manageable form.

Scientific notation is created by moving the decimal point in a number so that it lands after the first single non-zero digit. The number of places the decimal point is moved is represented as an exponent of the base 10.

Examples

Example 50:

The distance from the Earth to the Sun is approximately 149,600,000 kilometres. Write this distance using scientific notation.

Solution 50:

In the number 149,600,000 the decimal point is after the last zero on the right. For scientific notation, we move the decimal point 9 places to the left so that it lands between the 1 and the 4. In order to move the decimal point 9 places to the left, we multiply by 10^9.

$$149{,}600{,}000 = 1.496 \times 10^9$$

Example 51:

The diameter of an atom is approximately 0.0000000005 metres. Write this distance using scientific notation.

Solution 51:

This time we will need to move the decimal point to the right 10 places so that it lands after the 5. In order to move the decimal point 10 places to the right, we multiply by 10^{-10}:

$$0.0000000005 = 5 \times 10^{-10}$$

4. ALGEBRAIC RELATIONSHIPS

Definitions

Terminology:	Definition:
Variable	Usually a letter (or a symbol) that represents some unknown value. For example, *x, y, a, n.*
Term	Either a number on its own or the product of a number and a variable. For example, $3x$, $-5x^2y$, 12.
Variable term	A term that contains a variable or variables. For example, $3x$, $-2a^2b^3$.
Coefficient	The number part of a variable term that is multiplying the variable. For example, 5 in $5ab$ or -7 in $-7x$. If there appears to be no coefficient such as in the term x, it would be 1.
Constant term	A term that does not contain a variable. For example, 3 or -8.
Algebraic expression	A collection of variable and/or constant terms. Each term is separated by an addition or subtraction. For example, $3a^2b - 7ab + 2ab^2 - 8$.
Like Terms	Terms that contain exactly the same variable/s with exactly the same exponents on those variables. For example, $3x^2y$ and $-5x^2y$ are like terms whereas $-2x^3y^2$ and $-2x^2y^3$ are not like terms. The coefficients on like terms may be different.
Monomial	An algebraic expression containing a single term. For example, $-3x^5y$.
Binomial	An algebraic expression containing two terms. For example, 3a + 5b.
Trinomial	An algebraic expression containing three terms. For example, $x^2 - 3x + 5$.
Polynomial	Any algebraic expression. This includes monomials, binomials, and trinomials, but usually we reserve the word 'polynomial' for an expression containing more than three terms.
Degree	In an algebraic expression, the degree is the highest exponent or exponent sum. For example, the degree of the term $3a^4$ is 4, the degree of the term $-2m^3n^8$ will be the sum of 3 and 8 which is 11.

Simplifying Expressions

Definitions

An algebraic expression is simplified by combining **like terms** (see previous page for the definition of a like term). Once like terms have been identified, the coefficients are combined through addition or subtraction.

Examples

Note: By convention, expressions are usually ordered in terms of descending exponents starting with the highest degree. If there are more than one variable, the variables are placed in alphabetical order.

Example 53:

$$(x^2 + xy - 3y^2) - (2x^2 - 3xy + 5y^2) =$$

Solution 53:

The first step is to remove the brackets from the expression. In this particular question, the brackets are grouping two separate polynomials that are being subtracted from each other. When brackets are removed, pay attention to the subtraction of the second set of brackets as the subtraction will be distributed on each of the terms which will have the effect of changing the signs.

$$(x^2 + xy - 3y^2) - (2x^2 - 3xy + 5y^2) = x^2 + xy - 3y^2 - 2x^2 + 3xy - 5y^2$$

The like terms are then collected:

$$\begin{aligned} x^2 + xy - 3y^2 - 2x^2 + 3xy - 5y^2 &= x^2 - 5x^2 + xy + 3xy - 3y^2 - 5y^2 \\ &= -4x^2 + 4xy - 8y^2 \end{aligned}$$

Example 54:

$$3(x^2 - 5) + 2(x^2 + 3) =$$

Solution 54:

As with any expression (or equation) in math, **order of operations** must be observed. In this case, each bracket is being multiplied by a number which is distributed onto each term inside the bracket,

$$3(x^2-5)+2(x^2+3) = 3(x^2)+3(-5)+2(x^2)+2(3)$$
$$= 3x^2-15+2x^2+6$$

Next, like terms are collected:

$$3x^2-15+2x^2+6 = 3x^2+2x^2-15+6$$
$$= 5x^2-9$$

Identify Equivalent Expressions

Definitions

Recall that equivalent means writing an expression in a different, but equal, form.

Examples

Example 55:

Which of the following expressions is equivalent to $3(2x^2-5)-7(x^2-2)$?

A. $-x^2+1$
B. $-13x^2-29$
C. x^2-1
D. $-x^2-29$

Solution 55:

We simplify the expression given in the question by multiplying the brackets out then collecting like terms,

$$3(2x^2-5)-7(x^2-2) = 3(2x^2)+3(-5)+(-7)(x^2)+(-7)(-2)$$
$$= 6x^2-15-7x^2+14$$
$$= -x^2-1$$

We can see that the answer is A.

Creating and Evaluating Expressions

Creating Expressions

This topic requires and understanding of various terms used in mathematical operations and knowledge of how to translate these into an expression using algebra. At the end of this study guide is a glossary of the various terms used.

Examples

Example 56:

Which of the following expressions is 3 times *n* plus 5?

A. $3(n+5)$
B. $3n+5$
C. $3\times 5+n$
D. 3^n+5

Solution 56:

Most often, the way the sentence is read from left to right is the order in which we write it using algebra. So "3 times *n* plus 5" can be written as the expression, 3*n* + 5. So, answer B.

Note: if the sentence had said "3 times the sum of *n* plus 5", we would write that as $3(n+5)$.

Note: Beware of subtraction as in "7 less than n" which would be written as $n-7$.

Evaluating Expressions

The steps to follow in evaluating an algebraic expression are as follows:

1. Substitute the numerical values of variables into the algebraic expression placing brackets around the numerical values.
2. Evaluate the expression using the correct order of operations (see **1. Rational Numbers**)

Examples

Example 57:

Evaluate the expression a^2b-3ab^2+5, if $a=-2$ and $b=3$

Solution 57:

We substitute the values provided for the variable into the algebraic expression,

$$
\begin{aligned}
a^2b - 3ab^2 + 5 &= (-2)^2(3) - 3(-2)(5)^2 + 5 \\
&= (4)(3) + 6(25) + 5 \\
&= 12 + 150 + 5 \\
&= 167
\end{aligned}
$$

5. LINEAR APPLICATIONS AND GRAPHS

Definitions

A **linear equation** is usually presented in one of two forms:

Slope/y-intercept:

$$y = mx + b$$

where ‘m’ represents the slope (steepness and direction) of the line and ‘x’ represents the ‘y’ intercept (where the line crosses the ‘y’ axis)

Standard:

$$Ax + By = C \text{ where } A, B, C \text{ represent numbers}$$

The ‘x’ and ‘y’ planes represent the coordinate system that is used to graph various functions and relations such as linear equations. It consists of a horizontal number line or axis called the ‘x’ axis, and a vertical number line or axis called the ‘y’ axis.

The two axes intersect at the **origin**, which has the coordinate:

(0, 0)

Coordinates are ordered pairs of numbers that locate points on the ‘x’ and ‘y’ grid. The first number of a coordinate represents the ‘x’ coordinate, and the second number represents the ‘y’ coordinate.

Applying Linear Equations to Real-life contexts

Definitions

When linear equations are applied to real-life contexts, the slope usually represents some rate (see **2. Ratios and Proportional Relationships**) and the ‘y’ intercept represents a starting or initial value.

Examples

Example 58:

For a particular landscape project, labour is charged at $25 per hour and total materials cost $2500 If ‘h’ represents the number of hours to complete the project,

which of the following expressions represents the total cost of the project?

A. $25 + 2500h$

B. $2525h$

C. $25 \times h \times 2500$

D. $2500 + 25h$

Solution 58:

In the question, we are given two types of cost. One is a variable cost, the labour, and the other is the fixed cost, the material. In a linear equation, the variable cost means that it is dependent on a variable, in this case the number of hours, ‘h’. For linear equations, this represents a slope of 25. The fixed cost occurs regardless of the variable, so it represents the ‘y’ intercept, 2500. We can put this together in the slope ‘y’ intercept form of a linear equation as:

$$25h + 2500.$$

Considering the answers provided, this is the same as ‘D’.

Example 59:

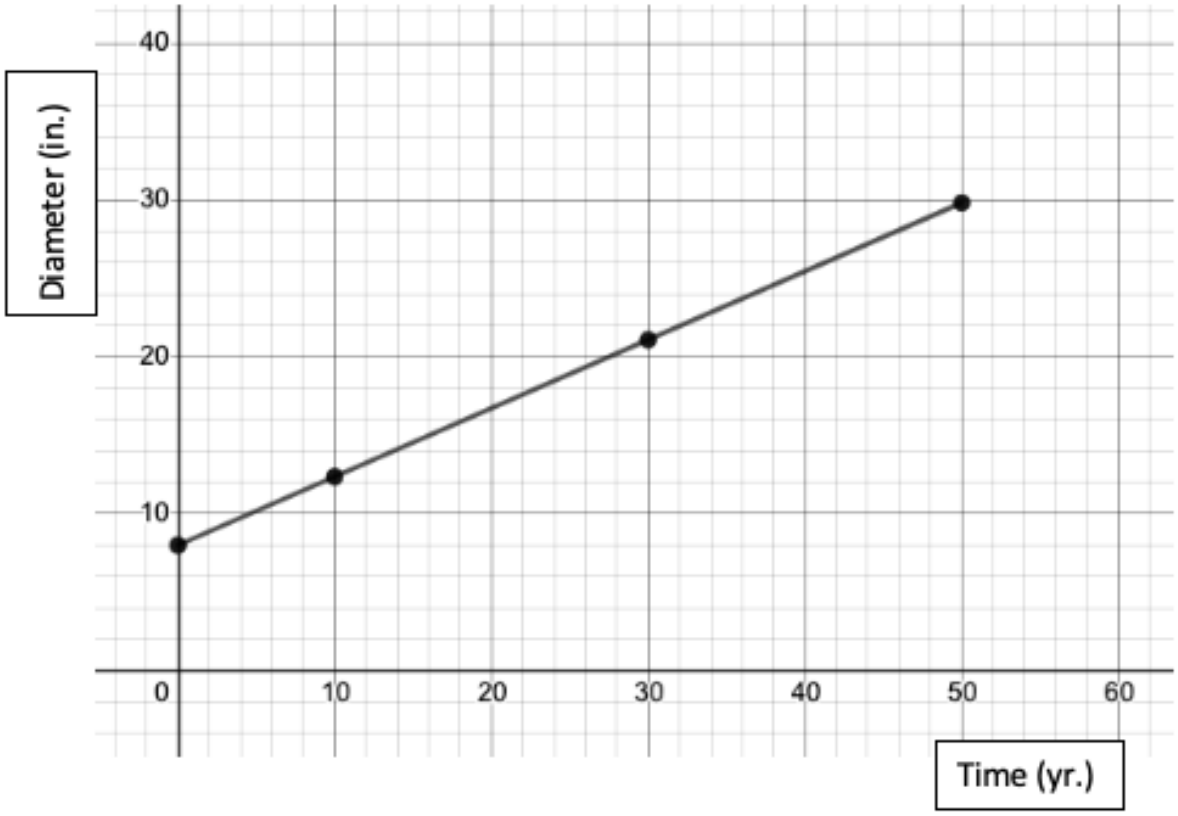
The graph below represents data collected in the growth of the diameter of a Fir tree over several years. Based on the pattern established in the graph, approximately how old would a tree be in years if it had a diameter of 32 inches?

A. 50

B. 55

C. 60

D. 22



Solution 59:

When looking at graphs it is important to take note what the axes represent and the scale being used to number the axes. The horizontal axis represents the time in years. Each square represents 2 years. The vertical axis represents the diameter of the Fir tree. Each square represents 2 inches.

To predict the age of the tree when it has a diameter of 32 inches is to extrapolate or extend the line that is drawn. By doing that, we can see that the tree will be approximately 55 years old. Therefore, the answer is ‘B’.

Using Elementary Linear Functions to describe relationships

Definitions

Slope, or ‘m’, is a measure of how steep a line is. It also indicates the direction of a line. If a line slopes up from left to right, we say it has a positive slope; if the line slopes down from left to right it has a negative slope.

The slope of a line is determined by calculating the amount of vertical rise, up or down, in relation to its horizontal run, moving to the right.

If we are given two points

$$(x_1, y_1) \text{ and } (x_2, y_2),$$

we can determine the slope using the formula:

$$slope = \frac{rise}{run} = \frac{y_2 - y_1}{x_2 - x_1}$$

Examples

Example 60:

What is an equation of the line passing through the points:

$$(0, 3) \text{ and } (2, 2)$$

Solution 60:

If we look carefully at the information given, we see that the point

$$(0, 3)$$

is in fact the ‘y’ intercept, or ‘b’. If we use the slope ‘y’ intercept form of a line, we can substitute 3 for ‘b’:

$$y = mx + b$$
$$y = mx + 3$$

To find ‘m’, the slope, we use the slope formula:

$$slope = \frac{y_2 - y_1}{x_2 - x_1} = \frac{(2) - (3)}{(2) - (0)} = \frac{-1}{2}$$

So, an equation the equation of the line is

$$y = -\frac{1}{2}x + 3$$

Graphing Linear Equations in Two Variables

Definitions

A graph gives are a visual interpretation of the relationship between 2 variables. This section provides a summary of graphing linear equations.

Examples

Example 61:

Graph the line

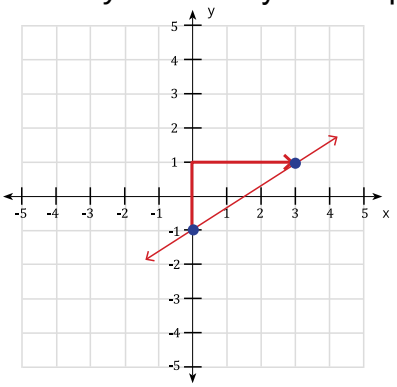
$$y = \frac{2}{3}x - 1$$

Solution 61:

Given the equation is in slope/y-intercept form, it is possible to see that the slope is two-thirds and the 'y' intercept is negative 1. One way to graph a line when it is in slope 'y' intercept form is to plot the 'y' intercept first, then use the slope in terms of rise and run to plot another point. For this equation, the slope is

$$\frac{2}{3}$$

so, we can say from the 'y' intercept the rise is up 2 and the run is across 3.



Parallel and Perpendicular Lines

Definition

Parallel lines stay exactly the same distance apart. For example, train tracks are parallel even when they go around a curve. If two linear equations are parallel, they will have the same slope.

Perpendicular lines are lines that meet at right angles, or 90 degrees. For example, a wall is usually perpendicular to the ceiling. The slopes of perpendicular lines are negative **reciprocals** of each other.

Note: the ‘y’ intercept, ‘b’, does not affect whether lines are parallel or perpendicular.

Examples

Example 62:

Which of the following lines are parallel to the line with the equation

$$y = -3x + 5?$$

A. $y = 3x + 5$
B. $y = -3x - 2$
C. $y = \frac{1}{3}x + 5$
D. $y = 5x + 3$

Solution 62:

All of the equations are given in the slope ‘y’ intercept format. Slope is given as the coefficient of ‘x’.

The equation of the line provided in the question, shown below, will have a slope of negative 3:

$$y = -3x + 5.$$

Looking at the answers;

A. $y = 3x + 5$ so the slope is 3,

B. $y = -3x - 2$ so the slope is -3

C. $y = \frac{1}{3}x + 5$ the slope is $\frac{1}{3}$ (this is the negative reciprocal of -3, so this line is in fact perpendicular to the line provided in the question),

D. $y = 5x + 3$, the slope is 5.

So, the answer is B.

6. LINEAR EQUATIONS

Definitions

A linear equation has a degree of 1 which means the highest exponent on any variable is 1.

Solving Linear Equations

Definitions

To **solve** a linear equation means to find all values of the unknown variable which will make the equation true (i.e. left side of the equation equals the right side of the equation).

A linear equation with one unknown variable usually has one solution (there can be alternate solutions, but these are not covered in this curriculum).

To solve a linear equation with one unknown variable, isolate the variable to one side of the equation by moving all the other terms and coefficients to the other side of the equation and simplifying.

To move a term or coefficient to the other side of the equation, do the opposite operation i.e. subtraction is the opposite of addition, division is the opposite of multiplication.

Examples

Example 63:

What is the solution to;

$$\frac{1}{2}(x - 4) + \frac{3}{4}(2x + 1) = 5?$$

Solution 63:

There can be several ways to solve equations, sometimes depending on the preferences of the solver. In this case, we could eliminate the fractions by multiplying everything by the lowest common denominator. For this problem the denominators are 2 and 4. The lowest common denominator of 2 and 4 is 4.

$$\frac{1}{2}(x-4)+\frac{3}{4}(2x+1)=5$$
$$4\left[\frac{1}{2}(x-4)\right]+4\left[\frac{3}{4}(2x+1)\right]=4[5]$$
$$2(x-4)+3(2x+1)=20$$

Next, we multiply out the brackets, simplify and solve the equation using opposite operations,

$$2x-8+6x+3=20$$
$$8x-5=20$$
$$8x-5+5=20+5$$
$$8x=25$$
$$\frac{8x}{8}=\frac{25}{8}$$
$$x=3\frac{1}{8}$$

Solving Systems of Two Linear Equations

Definitions

A system of equations is when there are 2 or more equations representing a certain situation. Typically, the solution to a system are the values of the variables that satisfy all the equations provided. Graphically, this is the point of intersection.

A common technique to solve a system of two linear equations is to use the **substitution method.**

Examples

Example 64:

$$2x-3y=8$$
$$x+y=9$$

The two lines given by the equations above intersect in the ‘x’ ‘y’ plane. What are coordinates of the point of intersection?

Solution 64:

Substitution is when one variable is replaced in terms of another. We look carefully at the equations provided and see in which equation it is easier to isolate one of the variables, ‘x’ or ‘y’. To isolate a variable is similar to solving equations using opposite operations. The second equation provided would seem a good choice where, in fact, it would be relatively straightforward to isolate for either variable. We’ll choose ‘x’:

$$\begin{aligned} x + y &= 9 \\ x + y - y &= 9 - y \\ x &= 9 - y \end{aligned}$$

Now we substitute, or replace, ‘x’ in the first equation with

$$9 - y,$$

$$\begin{aligned} 2x - 3y &= 8 \\ 2(9 - y) - 3y &= 8 \end{aligned}$$

Now we have an equation to solve with only one variable, ‘y’:

$$\begin{aligned} 2(9 - y) - 3y &= 8 \\ 2(9) + 2(-y) - 3y &= 8 \\ 18 - 2y - 3y &= 8 \\ 18 - 5y &= 8 \\ 18 - 5y - 18 &= 8 - 18 \\ -5y &= -10 \\ \frac{-5y}{-5} &= \frac{-10}{-5} \\ y &= 2 \end{aligned}$$

Now that we have a value for ‘y’, we now need to find the corresponding value for ‘x’. We can use the equation

$$\begin{aligned} x &= 9 - y \\ &= 9 - 2 \\ &= 7 \end{aligned}$$

The point of intersection is then $(7, 2)$.

Example 65:

At a college bookstore, Carla purchased a math textbook and a novel that cost a total of $54, not including tax. If the price of the math textbook is $8 more than 3 times the price of the novel, what is the price of a novel?

Solution 65:

For this type of question, it can be helpful to identify the variables. Let ‘m’ be the price of a math textbook and ‘n’ be the price of a novel. From the first sentence we will have,

$$m + n = 54$$

From the second sentence we will have,

$$m = 8 + 3n$$

This give us two equations to create a system to solve. First, use the second equation to substitute for ‘m’ in the first equation. Then simplify and solve for ‘n’.

$$\begin{aligned}
(8 + 3n) + n &= 54 \\
8 + 4n &= 54 \\
8 + 4n - 8 &= 54 - 8 \\
4n &= 46 \\
\frac{4n}{4} &= \frac{46}{4} \\
n &= 11.5
\end{aligned}$$

The price of a novel is $11.50

7. PROBABILITY AND SETS

Definitions

The probability of an **event** or events occurring is the likelihood of it happening.

Probabilities range from 0 to 1, or from

0% to 100%.

An **experiment** in probability helps to identify the boundaries within which we want to determine probabilities. For instance, an experiment could be rolling a 6-sided die (singular for dice) 100 times and recording what number the die has facing up.

An **event** is a particular occurrence within the experiment. A corresponding event for rolling a die 100 times could be the number of times a 6 is face up.

Defining Sample Spaces and Events using Set Notation

Definitions

A **sample space** in probability represents all the possible outcomes in an experiment. Often, we use curly brackets, { and }, to group all the outcomes.

Examples

Example 66:

For the experiment of rolling a die, what is the sample space?

Solution 66:

$$\{1, 2, 3, 4, 5, 6\}$$

Example 67:

For the experiment of tossing a coin twice, what is the sample space?

Solution 67:

If we let 'H' represent heads and 'T' represent tails, the sample space would be all the possible outcomes of this experiment,

$$\{HH, HT, TH, TT\}$$

Set Notation

A **set** is a group or collection of **elements**. Elements can be numbers, people, objects, characteristics, colours etc. Sample spaces and events are sets. Sets are usually bundled using curly brackets.

Set notation is used to apply certain operations to sets. The following is a list of operations:

Symbol:	Spoken as:	Description:	Example:
A, B, S, L etc.	Capital letters	The letter 'S' or 'U' is typically used for the sample space.	Let A be the set of even numbers: $A = \{2, 4, 6, 8\}$
A'	'A' complement	This operation lists all the elements that are not in set 'A' but still contained in the sample space	Let $S = \{1, 2, 3, 4, 5, 6, 7, 8\}$ and $A = \{2, 4, 6, 8\}$. Then $A' = \{1, 3, 5, 7\}$
$A \cup B$	'A' union 'B'	This operation unites or combines all elements in set 'A' with set 'B'	Let $A = \{2, 4, 6, 8\}$ and $B = \{2, 3, 5, \}$, then $A \cup B = \{2, 3, 4, 5, 6, 8\}$
$A \cap B$	'A' intersect 'B'	This operation takes only the elements that are in both set 'A' and set 'B'	Let $A = \{2, 4, 6, 8\}$ and $B = \{2, 3, 5\}$, then $A \cap B = \{2\}$
$(A \cap B)'$	The complement of 'A' intersect 'B'	Following order of operations, we find the intersection of set 'A' and 'B', then find the complement	Let $S = \{1, 2, 3, 4, 5, 6, 7, 8\}$, $A = \{2, 4, 6, 8\}$, $B = \{2, 3, 5\}$, then $(A \cap B)' = \{1, 3, 4, 5, 6, 7, 8\}$

Examples

Example 68:

$$P = \{4, 8, 16, 24, 31, 40\}$$
$$Q = \{4, 9, 16, 25, 36\}$$
$$R = \{3, 5, 7, 9\}$$

Sets ‘P’, ‘Q’ and ‘R’ are shown above. Determine the set that represents

$$P \cap (Q \cup R)$$

Solution 68:

Following order of operations, first determine inside the brackets,

$$Q \cup R,$$

Which combines the elements of set ‘Q’ with set ‘R’

$$Q \cup R = \{3, 4, 5, 7, 9, 16, 25, 36\}$$

Now find

$$P \cap (Q \cup R)$$

which represents the intersection of set ‘P’ with set

$$Q \cup R$$

$$P \cap (Q \cup R) = \{4, 16\}$$

Calculating Simple Probabilities

Definitions

The fundamental formula that is used to determine a simple (or single event) probability is:

$$P(A) = \frac{n(A)}{n(S)}$$

Where:

P represents probability
S is the sample space
A is the event we are interested in happening
$P(A)$ represents the probability of event A occurring
$n(A)$ represents the number of ways that event A can occur
$n(S)$ represents the total number of ways that any outcome can occur in the sample space

A probability can be written as a fraction, decimal or percentage (be comfortable converting between these three forms of numbers).

Examples

Example 69:

In a bag there are 3 red marbles, 8 blue marbles and 10 green marbles. If a marble is pulled out at random, what is the probability that it is a red marble?

Solution 69:

Let A be the event of pulling out a red marble.
There are a total of $3 + 8 + 10 = 21$ marbles in the bag so $n(S) = 21$. There are 3 red marbles so $n(A) = 3$.
Using the fundamental probability formula,

$$P(A) = \frac{n(A)}{n(S)} = \frac{3}{21} = \frac{1}{7}$$

Calculating Compound Probabilities

Definitions

A compound probability is when two or more events occur.

If the outcome of either of the two events do not affect each other, we say that they are **independent** events. For instance, the outcome of rolling a die does not affect the outcome of tossing a coin.

If two events, A and B, are independent, the probability that both will occur, $P(A\ and\ B)$ will be the product of the two separate probabilities, $P(A)$ and $P(B)$,

$$P(A\ and\ B) = P(A) \cdot P(B)$$

Note: it is possible to write $P(A\ and\ B)$ as $P(A \cap B)$.

Examples

Example 70:

What is the probability of getting the number 5 from rolling a die, and tossing a coin and getting a head?

Solution 70:

Firstly, we can see the outcome of each event does not affect the other, so they are independent. We then determine the probability of each event.

Let A be the event of getting a 5 from rolling a die, then $P(A) = \frac{1}{6}$. Let B be the event of getting a head when tossing a coin, then $P(B) = \frac{1}{2}$. We then determine the compound probability,

$$\begin{aligned} P(A\ and\ B) &= P(A) \cdot P(B) \\ &= \frac{1}{6} \cdot \frac{1}{2} \\ &= \frac{1}{12} \end{aligned}$$

Calculating Conditional Probabilities

Definitions

When the outcome of one event, A, does affect another event B, we say the events are **dependent**. For instance, if we pick a marble from a bag of marbles and not replace it in the bag, then this affects the probability of another marble being picked as the sample space has been reduced.

If two events, A and B, are dependent, the probability that both will occur, $P(A\ and\ B)$ will be the product of the following probabilities, $P(A)$ and $P(B\ given\ that\ A\ has\ occured)$,

$$P(A\ and\ B) = P(A) \cdot P(B\ given\ that\ A\ has\ occured)$$

Examples

Example 71:

In a bag there are 3 red marbles, 8 blue marbles and 10 green marbles. If two marbles are pulled out at random without replacement, what is the probability that they are both red marble?

Solution 71:

Consider the two marbles being pulled out as separate events. Let A be the event of pulling the first red marble and B the event of pulling the second red marble. Then $P(A) = \frac{3}{21} = \frac{1}{7}$. Given that event A has occurred and that red marble has not been replaced, the sample space has been reduced to $n(S) = 20$ and the number of red marbles left for event B is now $n(B) = 2$, so $P(B\ given\ that\ A\ has\ occured) = \frac{2}{20} = \frac{1}{10}$.

Now to calculate the probability that both A and B occur,

$$\begin{aligned} P(A\ and\ B) &= P(A) \cdot P(B\ given\ that\ A\ has\ occured) \\ &= \frac{1}{7} \cdot \frac{1}{10} \\ &= \frac{1}{70} \end{aligned}$$

8. DESCRIPTIVE STATISTICS

Definitions

Statistics is the collection and analysis of data.

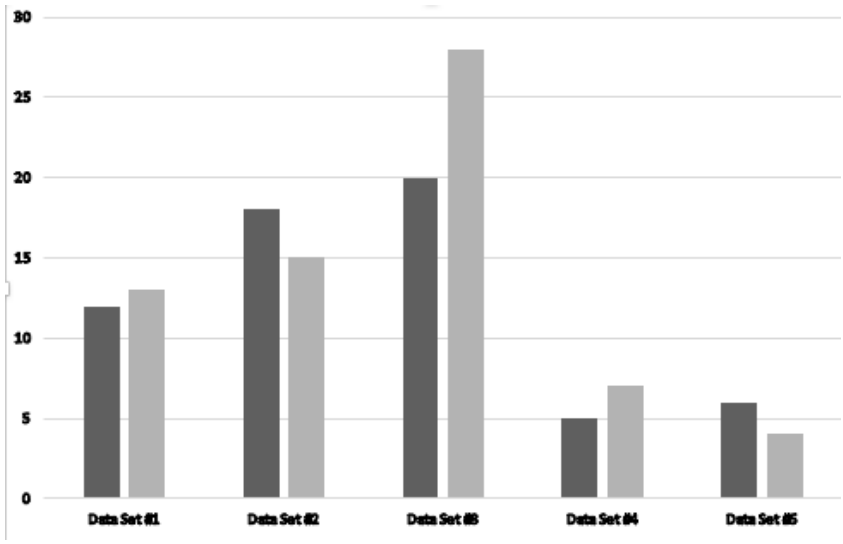
Descriptive statistics occurs after the data has been collected and is then graphed or calculations are made.

Interpreting Graphical Displays of Data

Histograms

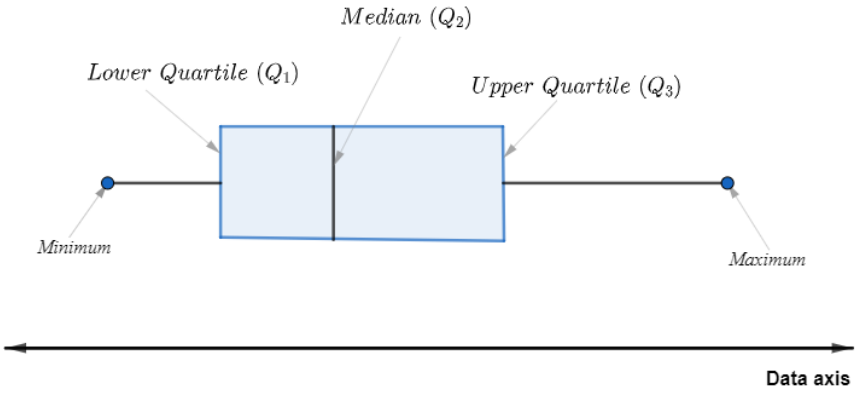
A histogram looks like a bar graph. Usually, the data range is on the horizontal axis and the frequency on the vertical axis.

The following graph shows double bars for each data set. This usually occurs when we want to represent growth or decline over a time period. The left-hand bar for each data set could represent one time period and the right-hand bar a subsequent time period. A legend accompanies a graph to interpret any colour (or shading) coding.



Box Plots

The following diagram represents a generalized horizontal box plot,



Across the bottom of the diagram is a horizontal data axis which represents the data being graphed. Above the data axis is a rectangular box. Its horizontal length is determined by the lower quartile and the upper quartile in line with the scaled data axis below. The median is drawn vertically within the box, again in line with its corresponding value on the data axis. The height of the box is drawn to big enough to make a box – there is no prescriptive measure for this. To the right of the box is a horizontal line starting at the middle of the left-hand side of the box and is drawn towards the left to the minimum value in the dataset. On the right of the box is another line starting at the middle of the right-hand side of the box and extending as far as the maximum value in the dataset.

In order to draw a box plot from a dataset, the following 5-number summary must be determined:

- the minimum

- the maximum
- the median
- the lower quartile
- the upper quartile

Quartiles (lower, median, upper) are the values that correspond to the dataset being divided into quarters.

The difference between the maximum and minimum data values is also called the **range.**

Calculating Measures of Centre

Definitions

To understand the characteristics of a dataset we are often interested in where the majority of the data is centred.

There are three measures of centre: mean, median and mode.

The **mean** is determined by adding up all the data and dividing by the number of data.

The **median** is the middlemost data after the data is organized from lowest to highest.

The **mode** is the most frequent data. Note: There can be more than one mode.

Examples

Example 72:

Determine the mean, median and mode for the data:

3, 8, 17, 10, 8, 12, 5, 14

Solution 72:

The mean is the sum of all the data divided by the number of data,

$$\frac{3+8+17+10+8+12+5+14}{8} = \frac{77}{8} = 9.625$$

For the median, first we sort the data from lowest to highest,

$3, 5, 8, 8, 10, 12, 14, 17$

With an even number of data, we see that two data are in the middle, in this case we have 9 and 10. We take the mean of these two data,

$$\frac{8+10}{2} = \frac{18}{2} = 9$$

The mode is the most frequent data. In this sample of numbers 8 is the mode as it occurs twice.

9. GEOMETRY CONCEPTS FOR PRE-ALGEBRA

Definitions

Perimeter: distance around the outside of a two-dimensional shape. The perimeter of a circle is called the **circumference**.

Area: region within a 2-dimensional shape (such as a triangle, rectangle, circle, etc.)

Volume: space within a 3-dimensional space (such as a box, sphere, cone, etc.)

Prism: This is a three-dimensional object that has the two identical ends and flat sides which are usually rectangles. The shape of the ends give the prism a particular name. For instance, a triangular prism has triangles as the end shapes and 3 rectangles for the sides.

Triangle: a 3-sided shape. A right-angled triangle contains one right angle (or $90°$).

Square: a 4-sided shape with all sides the same length meeting at $90°$.

Rectangle: a 4-sided shape with two pairs of sides of different lengths. Sides meet at $90°$.

Circle: a perfectly round shape created by connecting all points that are same distance from a fixed point called the centre. The distance from the centre to the circumference is called the **radius**. The **diameter** of a circle is a straight line drawn from one point on the circumference to another point on the circumference passing through the centre of the circle. The diameter is twice the radius, or

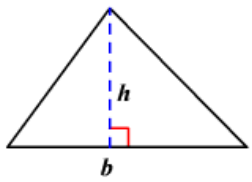
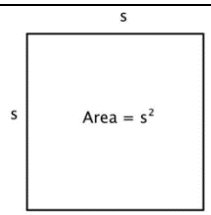
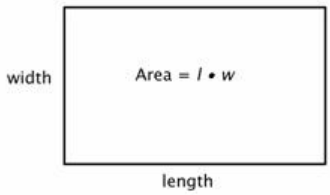
$d = 2r.$

Determining Perimeter and Area

Definitions

Given the perimeter is the distance around the outside of a shape, we can usually just add the various lengths together.

The following table gives the formulas for the areas of triangles, squares and rectangles.

Shape	Diagram	Formula
Triangle		$A = \frac{bh}{2}$ where b is the base, and h is the height of the triangle, perpendicular (at $90°$) to the base
Square		$A = s^2$
Rectangle		$A = l \cdot w$

Examples

Example 73:

Given the area of a triangle is 12 cm^2 with a base of 3 cm, determine the height of the triangle.

Solution 73:

The formula for the area of a triangle is $A = \frac{bh}{2}$. Given the area is 12 cm^2 and the base is 3 cm we can substitute these into the formula to find the height,

$$\begin{aligned} A &= \frac{bh}{2} \\ 12 &= \frac{3h}{2} \\ 12 \times 2 &= \frac{3h}{2} \times 2 \\ 24 &= 3h \\ \frac{24}{3} &= \frac{3h}{3} \\ 8 &= h \end{aligned}$$

So, the height of the triangle is 8 cm.

Circle Circumference and Area

Definitions

The circumference, C, of a circle with radius r or diameter d is determined using either of the following two formulas:

$$C = 2\pi r \qquad \text{or} \qquad C = \pi d$$

The area, A, of a circle with radius r is determined by the formula:

$$A = \pi r^2$$

Note: π represents an irrational number and is approximately 3.1415926. For some problems we leave π in the answer to be more exact.

Examples

Example 74:

For a circle with radius 3 cm, determine its circumference. Leave your answer in terms of π.

Solution 74:

Using the formula for circumference,

$$
\begin{aligned}
C &= 2\pi r \\
&= 2\pi(3) \\
&= 6\pi
\end{aligned}
$$

The circumference will be 6π centimetres

Example 75:

For a circle with diameter 7 cm, determine its area. Leave your answer in terms of π.

Solution 75:

Using the formula for area, we will first need to determine the radius of the circle. Given the diameter is 7 cm, the radius will be half the diameter, 3.5 cm,

$$
\begin{aligned}
A &= \pi r^2 \\
&= \pi(3.5)^2 \\
&= \pi(12.25) \\
&= 12.25\pi
\end{aligned}
$$

The area will be 12.25π centimetres^2.

Volume of Prisms

Definitions

Recall that a prism is a three-dimensional object that has the two identical ends and flat sides which are usually rectangles. Volume is the amount of space contained in a three-dimensional object.

Examples

Example 76:

A triangular prism has two identical triangles at either end, or 3 rectangles for the sides. The base of one triangle is 3 cm, and its height is 5 cm. The length of the prism is 10 cm. What is the volume of the prism?

Solution 76:

The general formula for the volume of a prism is given by,

$$V_{prism} = (area\ of\ end) \times (length\ or\ height)$$

For a triangular prism, we can write this formula as,

$$V_{triangle\ prism} = \frac{bh}{2} \times length$$

For the prism in question,

$$\begin{aligned} V_{triangle\ prism} &= \frac{(3)(5)}{2} \times 10 \\ &= 75 \end{aligned}$$

The volume of the prism is 75 cm^3.

10. GEOMETRY CONCEPTS FOR ALGEBRA 1

Creating Expressions for Area, Perimeter and Volume

Definitions

Here, we combine topics covered in **9. Geometry Concepts for Pre-Algebra** along with those in **4. Algebraic Relationships**.

Examples

Example 77:

If the length of a rectangle is $4x$ and the width is $2x + 1$, determine an expression for the perimeter of the rectangle.

Solution 77:

The perimeter of any shape is the distance around the shape. For the perimeter of the rectangle given in the question we would add,

$$\begin{aligned}\text{Perimeter} &= (4x) + (2x + 1) + (4x) + (2x + 1)\\ &= 12x + 2\end{aligned}$$

Example 78:

If the length of a rectangle is $4x$ and the width is $2x + 1$, determine an expression for the area of the rectangle.

Solution 78:

The area of a rectangle is given by the length times the width,

$$\begin{aligned}\text{Area} &= (4x)(2x + 1)\\ &= (4x)(2x) + (4x)(1)\\ &= 8x^2 + 4x\end{aligned}$$

Example 79:

For a rectangular prism the length is $4x$, width is $2x + 1$ and height is $3x$. Determine an expression for the volume of this prism.

Solution 79:

The volume of a rectangular prism is given as length times width times height,

$$\begin{aligned}\text{Volume} &= (4x)(2x+1)(3x)\\ &= \big((4x)(2x)+(4x)(1)\big)(3x)\\ &= (8x^2+4x)(3x)\\ &= (8x^2)(3x)+(4x)(3x)\\ &= 24x^3+12x^2\end{aligned}$$

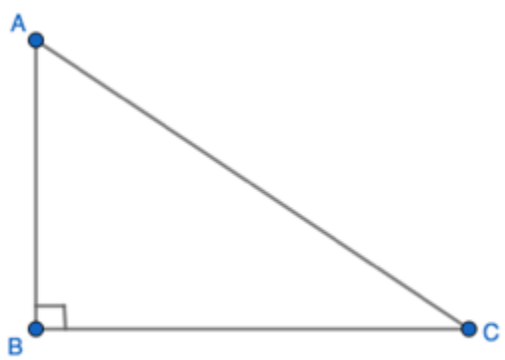
Using Pythagorean Theorem and Distance Formula

The Pythagorean Theorem

The Pythagorean Theorem is used to determine the length of a side in a right-angled triangle when the other two sides are known.

In words the Theorem says, the square on the hypotenuse is equal to the sum of the squares on the two shorter sides. The hypotenuse in a right-angled triangle is always the longest side, opposite the right angle.

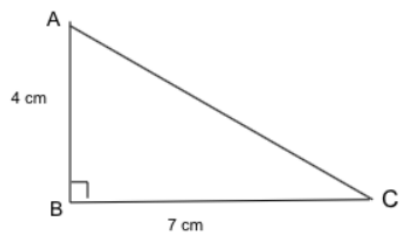
The diagram below shows a right-angled triangle ABC, where the angle at B is $90°$ as indicated by the small square. By definition, the hypotenuse is the side AC, across from the right angle, and the two shorter sides are AB and BC.



Examples

Example 80:

The diagram below shows a right-angled triangle ABC, where side $AB = 4$ cm, $BC = 7$ cm. Find the length of the side AC to the nearest hundredth. (calculator allowed)



Solution 80:

AC is the hypotenuse so applying the Pythagorean Theorem,

$$\begin{aligned} AC^2 &= AB^2 + BC^2 \\ &= (4)^2 + (7)^2 \\ &= 16 + 49 \\ &= 65 \end{aligned}$$

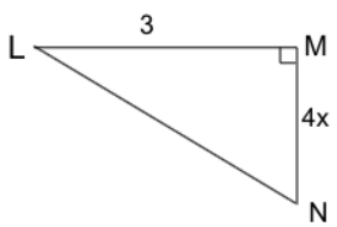
To find AC we now take the square root of both sides of the equation,

$$\begin{aligned} \sqrt{AC^2} &= \sqrt{65} \\ AC &= 8.06 \end{aligned}$$

So AC is 8.06 centimetres to the nearest hundredth.

Example 81:

The diagram below shows a right-angled triangle, where side $LM = 3$, $MN = 4x$. Find an expression for the length of the side LN.



Solution 81:

LN is the hypotenuse for this triangle. Applying the Pythagorean Theorem,

$$\begin{aligned} LN^2 &= LM^2 + MN^2 \\ LN^2 &= (3)^2 + (4x)^2 \\ LN^2 &= 9 + 16x^2 \\ \sqrt{LN^2} &= \sqrt{9 + 16x^2} \\ LN &= \sqrt{9 + 16x^2} \end{aligned}$$

Distance Formula

We can use the Pythagorean Theorem when we want to determine the distance between two points in the 'x' and 'y' planes.

Given two points (x_1, y_1) and (x_2, y_2), the formula used to determine the distance, d, between the two points is given by,

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

This formula is a variation of the Pythagorean Theorem.

Examples

Example 82:

Determine the distance between the two points $(-3, 3)$ and $(5, -1)$.

Solution 82:

Let the first point $(-3, 3)$ be (x_1, y_1), and the second point $(5, -1)$ be (x_2, y_2).
Then we apply the distance formula,

$$\begin{aligned} d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{\big((5) - (-3)\big)^2 + \big((-1) - (3)\big)^2} \\ &= \sqrt{(8)^2 + (-4)^2} \\ &= \sqrt{64 + 16} \\ &= \sqrt{80} \\ &= 4\sqrt{5} \end{aligned}$$

(See Simplifying Radicals under Calculate with Radicals in **3. Exponents** for how to simplify the radical.)

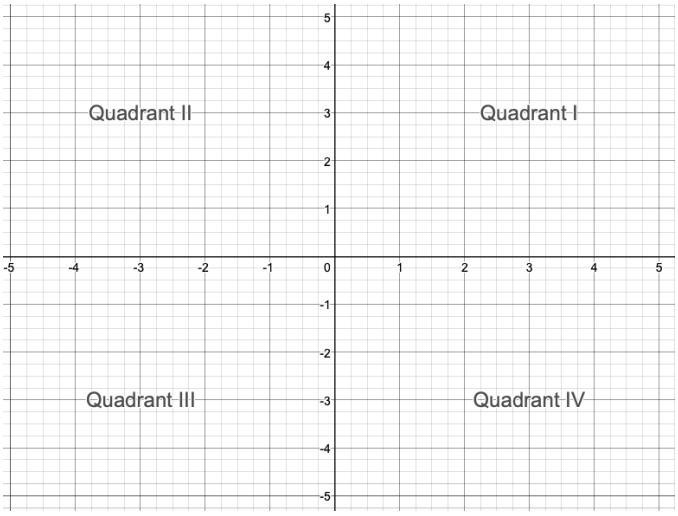
Evaluating Basic Geometric Transformations

Definitions

In the 'x' and 'y' plane, we will look at 2 basic transformations: Reflections and Rotations

Points are located in the 'x' and 'y' plane using coordinates. These points are typically labelled using a capital letter. For instance, the point with coordinates $(2, 3)$ might be labelled A. If a point is transformed in the 'x' and 'y' plane, it may move to another location. We use the notation A' ("A dash" or "A prime") to indicate a point that originally started off at A but has now been transformed.

It is helpful to use quadrant names to identify where points are located as illustrated by the following diagram of the ‘x’ and ‘y’ plane,



Quadrant I is in the top right quarter where both the ‘x’ and ‘y’ coordinates are positive.
Quadrant II is in the top left quarter where the ‘x’ coordinate is negative and the ‘y’ coordinate is positive.
Quadrant III is in the bottom left quarter where both the ‘x’ and ‘y’ coordinates are negative.
Quadrant IV is in the bottom right quarter where the ‘x’ coordinate is positive and the ‘y’ coordinate is negative.

A **reflection** is similar to the action a reflection has in a mirror. Typically, the ‘x’ or ‘y' axis are used as lines of reflection. This means that if point A starts in quadrant I and is reflected across the ‘x’ axis it will end up in quadrant IV as point A'.

The following chart illustrates where various points end up after reflections across the ‘x’ axis or ‘y’ axis:

Point	**Reflection across ‘x’ axis**	**Reflection across ‘y’ axis**
$A(3,4)$	$A'(3,-4)$	$A'(-3,4)$
$B(-5,2)$	$B'(-5,-2)$	$B'(5,2)$
$C(-7,-6)$	$C'(-7,6)$	$C'(7,-6)$
$D(3,-9)$	$D'(3,9)$	$D'(-3,-9)$

A **rotation** is when an object is turned around a fixed point. The fixed point is usually the origin. The number of degrees determines the amount of rotation. Rotations are often $90°$ or $180°$ in a clockwise or counter-clockwise direction.

The following chart illustrates where various points end up after reflections of $90°$ or $180°$ around the origin in a clockwise direction:

Point	Rotation clockwise $90°$	Rotation clockwise $180°$
$A(3,4)$	$A'(4,-3)$	$A'(-4,-3)$
$B(-5,2)$	$B'(2,5)$	$B'(5,-2)$
$C(-7,-6)$	$C'(-6,7)$	$C'(7,6)$
$D(3,-9)$	$D'(-9,-3)$	$D'(-3,9)$

Note: When doing problems related to geometric transformations it is often helpful to sketch out a graph with 4 quadrants, locate the point in question, then consider where the point will end up after the transformation/s.

Examples

Example 83:

Point Q lies in the xy-plane. The coordinates of Q are $(-1,4)$. The point Q is reflected across the x-axis and then rotated $180°$ clockwise about the origin. What are the coordinates of the Q'?

A. $(4,1)$
B. $(-1,-4)$
C. $(-1,4)$
D. $(1,4)$

Solution 83:

Reflecting a point across the x-axis will cause the y-coordinate of the point to change sign to $(-1,-4)$. The point is now in quadrant III. The rotation $180°$ clockwise about the origin will now transform the point to quadrant I. This could be answer A. or D. The effect of a $180°$ rotation keeps the x- and y-coordinates. Given that we are now in quadrant I, the point Q' will be $(1,4)$. Answer D.

Appendix A: Glossary

Absolute Value: the magnitude of a quantity
Algebraic expression: a statement containing numbers, variables and mathematical operation signs
Base (exponents): the term that the exponent is attached to
Binomial: an algebraic expression with two terms related to each other by a mathematical operation; (for example, $7x^3 + 9$)
Circumference: the perimeter of a circle
Coefficient: the number multiplied by a variable or variables in an algebraic term; (for example, 3 is the coefficient in $3x^2y$)
Constant term: a term in a simplified algebraic expression that contains no variables and thus never changes; (for example, 4 is the constant term in the algebraic expression, $5x^3 - 18x + 4$)
Cube root: the value of the number, which multiplied by itself three times gives the original number
Degree (of a term**):** the sum of exponents on all the variables in a term
Denominator: the bottom part of a fraction or rational expression
Dependent (probability): when the outcome of an event affects the outcome of another
Diameter: the line across a circle from one point on the circumference to another, passing through the centre
Difference: the result of subtraction
Elements (probability): the objects or items in a set
Equation: an equation uses an equal sign to state that two expressions are the same or equal to each other; (for example, $5x^2 + 18 = 25$)
Equivalent: math forms that represent the same value or quantity. For example, 0.5, $\frac{1}{2}$ and 50% all equivalent representations of the same quantity
Evaluate: to calculate the numerical value
Even: numbers that are divisible by 2 are considered to be even; these are 2, 4, 6, 8 and numbers that end in 0, 2, 4, 6, or 8
Expand: to eliminate brackets in an algebraic expression using the correct method of expanding
Event (probability)**:** a particular outcome in the sample space
Experiment (probability): used to determine probabilities, it provides the collection of data necessary for statistical probabilities
Exponent: the number in a power indicating how many times repeated multiplication is done
Factor: a number and/or variable that will divide into another number and/or variable exactly (for example, factors of $6x^2$ are x, x^2, 1, 2, 3, and 6)
Fraction: a rational number representing part of a whole

Greatest common factor: the greatest factor (consisting of numbers and/or variables) that ALL the terms in a given algebraic expression are divisible by; (for example, the greatest common factor of $27x^3y$ and $36xy^2$ is $9xy$)

Improper fraction: when the numerator is larger than the denominator

Independent (probability): when the outcome of one event does not affect another

Integer: a positive or negative whole number

Irrational number: a number that cannot be expressed as a fraction e.g. $\pi, \sqrt{2}$

Like terms: terms with the same variables AND exponents on those variables; the coefficients of like terms may be different; (for example, $4y^3x$ and $-9xy^3$ are like terms, but not $10y^2x$)

Linear equation: an equation with degree 1, that is, 1 is the highest exponent on any one variable (for example, $3x + 2 = 10$)

Monomial: an algebraic expression with one term; (for example, $7x^3$)

Mixed numbers (or fractions): a whole number and a proper fraction (for example, $5\frac{2}{3}$)

Numerator: the top part of a fraction or rational expression

Origin: where the axes intersect for the xy-plane, $(0,0)$

Perfect square: a number whose square root is a whole number and/or a variable with an even exponent

Perimeter: the total distance or length around the outside of a shape

Polynomial: an algebraic expression with many terms related to each other by mathematical operations; (for example, $7x^3 + 5x^2 + 18xy - 9$)

Power: a number raised to an exponent

Product: the result of multiplication

Proper fraction: when the numerator is smaller than the denominator

Proportion: the equality of two ratios or two rates

Quartile (statistics): data is first organised from lowest to highest, then the median (Q_2) is the middlemost datum which splits the data in half, the lower quartile (Q_1) splits the lower half of the data in half, the upper quartile (Q_3) splits the upper half in half

Radical expression: an expression that has a square root, $\sqrt{\square}$, cube root, $\sqrt[3]{\square}$ or any n^{th} root, $\sqrt[n]{\square}$

Radicand: the number or variable(s) that is/are beneath the radical sign; (for example, 5 is the radicand in $\sqrt[3]{5}$)

Radius: the line from the centre of a circle to the circumference

Range (statistics): the difference between the maximum and minimum data values of a sample

Rate: a comparison of two or more quantities with different units

Ratio: a comparison of two or more quantities with the same units

Rational number: a number written in the form $\frac{a}{b}$ where a, b are integers b is not 0

Rational expression: a fraction where the denominator is an algebraic expression
Reciprocal: the multiplicative inverse of a number; the product of two reciprocals is by definition equal to 1. For a fraction $\frac{a}{b}$, the reciprocal is $\frac{b}{a}$.
Sample space (probability): the set from which events can occur
Set (probability): a group or collection of objects (elements)
Simplify: to write an algebraic expression in simplest form; an expression is in simplest form when there are no more like terms that can be combined
Solve: to find the numerical value
Squared: raised to the exponent 2
Square root: the square root of a number is the value of the number, which multiplied by itself gives the original number
Substitution method: used in solving linear systems where one variable is replaced in terms of the other variable
Sum: the result of addition
Trinomial: an algebraic expression with three terms related to each other by mathematical operations; (for example, $7x^3 + 5x - 9$)
Unit Rate: a rate when the second quantity is single (for example, 50 km/hr)
Variable: a letter of the alphabet used to represent an unknown number or quantity
Variable term: a term in a simplified algebraic expression that contains variables; (for example, $5x^3$ and $-18x$ are variable terms in the algebraic expression, $5x^3 - 18x + 4$)
xy-plane or 'x' 'y' plane: the Cartesian coordinate system