Solving Systems of Linear Equations by Elimination

Note: There are two Solving Systems of Linear Equations handouts, one by Substitution and another by Elimination.

A system of linear equations involves one or more equations working together. **This handout focuses on solving systems of linear equations with one solution.** These systems are known as "consistent and independent" with one point of intersection.

Note: A linear equation of the form Ax + By + Cz = D, where a, b, c are numbers forms a plane in three-space.

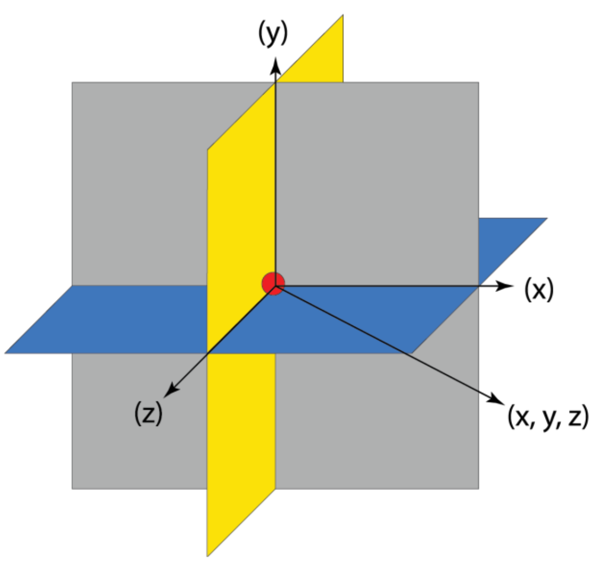
Example 1:

Given the linear equations;

1) $x+2y-3z = 17$
2) $-3x+y-7z = 0$
3) $5y+12z = 13$

Solve for the values of x, y, and z by **elimination**.

In this example, we have three **Linear Equations** and Three **Unknown Variables** in a Three **Dimensional-Space**



We will get **One Solution** of the form **(x, y, z)**

Solution with steps:

Step 1:

Stack the equations vertically and line up their variables. Number them 1), 2) and 3).

1) x+2y-3z = 17
2) -3x+y-3z = 17
3) 5y = 12z = 13

Step 2:

Multiply one (or two) of the equations by a number that will help you eliminate a variable when both equations are added together.
Let's multiply equation **(1) by 3** and then add it to equation **(2)** to eliminate x**.**

1) x3 = 3x + 6y - 9z = 51

$$\begin{array}{l} \mathbf{3x + 6y \ - 9z \ = 51} \\ + \ (-3x + \ y \ - 7z \ = 0) \\ \hline \mathbf{4)} \qquad 7y - 16z = 51 \end{array}$$

Step 3:

Now we have two equations **3)** and **4)** with only two variables instead of three.

Thus, we need to repeat the process of *elimination* until we can solve for the value of at least one unknown variable.

Let's multiply equation **(3) by 7** and multiply equation **(4) by -5**, and then add both equations to eliminate the **y** variable.

3) 5y + 12z = 13
4) 7y – 16z = 51
3) x 7 = **35y + 84z = 91**

$$\begin{array}{r} \mathbf{-35y \ + 80z \ = \ -255} \\ + \ (\mathbf{35y \ + 84z \ = 91}) \\ \hline 164z = \ -164 \end{array}$$

Now we can solve for z.

164z = 164

z = -1

Step 4:

We now know the value of z. Let's substitute **z = -1** into equation **(3)** to obtain the value of **y**.

3) 5y + 12z = 13

5y + 12(-1) = 13

$$y = \frac{(13 + 12)}{5}$$

y = 5

Step 5:

Now we have the values of **y and z.** Let's substitute the values of **y and z** into equation **(1)** to obtain the value of **x**.

1) x + 2y – 3z = 13

2) x + 2(5) – 3(-1) = 17

x = 17 – 10 - 4

x = 4

Step 6:

Solution: (**x**, **y**, **z**) **= (4 ,5, -1**)

Step 7:

Check! To make sure our values for **x, y and z** are correct, let's substitute all the values into equation **(2)** and see if it holds true.

1) -3x + y – 7z = 0

-3(4) + (5) – 7(-1) = 0

0 = 0

L.H.S = R.H.S

Example 2:

Given the linear equations,

1) **$3x + 4y - z = 6$,**
2) **$4x - 5y + 2z = 4$** and
3) **$2x + 2y - z = 1$**
solve for the values of **x, y, and z** by **elimination**.

Solution with steps:

Step 1:

Stack the equations vertically and line up their variables. Number them 1), 2) and 3).

1) $3x + 4y - z = 6$
2) $4x - 5y + 2z = 4$
3) $2x + 2y - z = 1$

Step 2:

Multiply equation **(3) by 2** and add it to equation **(2)** to eliminate **z**.

1) x 2 = **$4x + 4y - 2z = 2$**

$$\begin{array}{rl} \mathbf{4x + 4y - 2z} & \mathbf{= 2} \\ + \ (4x - 5y + 2z & = 4) \\ \hline \mathbf{4)} \ 8x - y & = 6 \end{array}$$

Step 3:

Since we still have two unknown variables, we'll need to do this process again with another set of two equations (where **z** is also eliminated).
Let's multiply equation **(1) by 2** and then add it to equation **(2).**

1) x 2 = **$6x + 8y - 2z = 12$**

$$\begin{array}{rl} \mathbf{6x + 8y - 2z} & \mathbf{= 12} \\ + \ (4x - 5y + 2z & = 4) \\ \hline \mathbf{5)} \ 10x + 3y & = 16 \end{array}$$

Step 4:

Now we have two equations **4)** and **5)** with only two variables instead of three.

Let's multiply equation **(4) by 3** and add it to equation **(5)** to eliminate **y**.

4) $8x - y = 6$
5) $10x + 3y = 16$

3) $8x - y = 6$

$$\begin{array}{rl} \mathbf{24x - 3y} & \mathbf{= 18} \\ + \; (10x + 3y & = 16) \\ \hline 34x & = 34 \end{array}$$

Now we can solve for x:
$34x = 34$
x =1

Step 5:

Substitute this **x** value into equation **(4)** to obtain the value of **y**.

2) $8x - y = 6$
$8(1) - y = 6$
y = 2

Step 6:

Substitute both the **x and y** value into equation **(3)** to obtain the value of **z**.
3) $2x + 2y - z = 1$
$2(1) + 2(2) - z = 1$
z = 5

Step 7:

Solution
(**x**, **y**, **z**) = (**1**, **2**, **5**)

Step 8:

Check! To make sure our values for **x, y, and z** are correct, let's substitute all the values into equation **(1)** and see if it holds true.

$3x + 4y - z = 6$
$3(1) + 4(2) - (5) = 6$
$6 = 6$
L.H.S = R.H.S

Steps to solve by Elimination:

Step 1:	To solve for a consistent system, check to see if the number of equations is equal to the number of unknown variables.
Step 2:	Stack the equations vertically and line up their variables. Number the equations 1), 2) and 3).
Step 3:	Multiply at least one equation by a value such that when two equations are added together, one of the unknowns is eliminated. (We may need to repeat this process depending on the number of unknowns involved.)
Step 4:	Once we have the actual value of a variable, we substitute this value into one of the equations to get the value of the other variables.
Step 5:	Check! Once you have found the values of the variables substitute them into one of the equations and simplify. If the left hand side of the equation is equal to the right hand side then you are done.

EXERCISES:

Solve the following system of linear equations:

1) $2x + 3y - z = 7$
   $x + 4y - 2z = 5$
   $3z + 3x = 15$

2) $4x - 2y + z = -1$
   $5x + 3y - 2z = 21$
   $2x - 5y + 3z = -16$

3) $2w - x + y + 3z = 24$
   $w + 3x + 2y - z = 15$
   $4y - 5w = 10$
   $5x + 2z = 27$
   (Hint: Eliminate two common variables.)

SOLUTIONS:

1) $x = 1, y = 3, z = 4$
2) $x = 2, y = 7, z = 5$
3) $w = 2, x = 3, y = 5, z = 6$