Solving Linear Equations in One Variable

A linear equation is an algebraic equation with a degree of 1. This means that the highest exponent on any variable in the equation is 1.

A **linear equation in one variable** can be written in the form ***ax +b = c***, where *a*, *b*, and *c* are real numbers.

General guidelines for solving linear equations in one variable:

- Simplify anything inside brackets.
- Get rid of any brackets using the distributive property:

  $a(b+c) = ab + ac$

- Collect like terms.
- Isolate the unknown variable by moving all other terms to the other side of the equation. To move a term across the equal sign, do the *opposite* operation on the other side. (Addition → subtraction; subtraction → addition; multiplication → division; division → multiplication).
- Linear equations can be first simplified using cross multiplication.

  $\frac{a}{b} = \frac{c}{d}$ can be written as $ad = bc$.

Note: The above are *guidelines* only and are NOT a step-by-step guide to solving linear equations in one variable. Different equations will require different techniques for solving. In many cases, there is more than one way to solve a linear equation.

Examples: Solve for x.

Example 1:

$2(x + 1) = 3(4 – x)$

- Expand the brackets

  $2x + 2 = 12 – 3x$

- Move the x terms to the same side and the numbers to the other side.

  $2x + 3x = 12 – 2$

- Collect like terms.

  $5x = 10$

- Move 5 to the other side by dividing by 5 on the right side.

  $x = \frac{10}{5}$

- Do the division.

  $x = 2$

Example 2:

$\frac{2}{3}x - \frac{1}{2}x = \frac{4}{3}$

- Move the like terms to the same side of the equation.

  $\frac{2}{3}x - \frac{4}{3} = \frac{1}{2}$

- In order to subtract the x terms, find a common denominator and re-write both terms as equivalent fractions with the same denominator.

  $\frac{4}{6}x - \frac{3}{6}x = \frac{4}{3}$

- Subtract like terms.

$$\frac{1}{6}x = \frac{4}{3}$$

- Move 6 to the other side by multiplying by 6 on the right side.

$$x = \frac{4}{3}(6)$$

- Do the multiplication.

$$x = \frac{24}{3}$$

- Reduce the fraction.

x = 8

Example 3:

$$\frac{3}{x} + 4 = 2$$

- Move 4 to the right side of the equation.

$$\frac{3}{x} = 2 - 4$$

- Subtract.

$$\frac{3}{x} = -2$$

- Cross multiply.

$$\frac{3}{x} = \frac{-2}{1}$$

- Move the x term to the right side of the equation by doing the opposite operation.

3 = -2x

- Leave your answer as an improper fraction or change it to a mixed number.

$$x = -\frac{3}{2} \; or \; x = -1\frac{1}{2}$$

Example 4:

$$\frac{3x-1}{5} = 3x + 1$$

- Cross multiply to get rid of fractions on both sides of the equation. (Note that 3x + 1 can be written as a fraction with a denominator of 1).

  $3x - 1 = 5(3x + 1)$

- Expand the brackets on the right side using the distributive property.

  $3x - 1 = 15x + 5$

- Move 15x to the left side and the number terms to the right side of the equation.

  $3x - 15x = 5 + 1$

- Collect like terms.

  $-12x = 6$

- Move the number to the right side of the equation by doing the opposite operation, leaving x term alone.

  $$x = -\frac{6}{12}$$

- Simplify the fraction by reducing to lowest terms.

  $$x = -\frac{1}{2}$$

Example 5:

$$\frac{2x+1}{3}+\frac{1}{2}=1-\frac{x-3}{5}$$

- To get rid of fractions on both sides of the equation, first find the lowest common multiple (LCM) of the denominators.

  $$\frac{2x+1}{3}+\frac{1}{2}=1-\frac{x-3}{5}$$

- Multiply **every** term in the equation by the LCM. In this case, 30.

  $$30\left(\frac{2x+1}{3}\right)+30\left(\frac{1}{2}\right)=30(1)-30\left(\frac{x-3}{5}\right)$$

- Simplify each term. Fractions can be reduced

  $10(2x + 1) + 15(1) = 30 – 6(x - 3)$

- Get rid of brackets by using the distributive property.

  $20x + 10 + 15 = 30 – 6x + 18$

- Move like terms to the same side of the equation.

  $20x + 6x = 30 + 18 – 10 – 15$

- Calculate like terms

  $26x = 23$

- Move 26 to the right side of the equation by doing the opposite operation.

  $$x=\frac{23}{26}$$

Practice Questions:

Solve for x.

a) $3-4x=8x+3$

b) $20x + 4x - 18 = 30 - 6x$

c) $1 - (2x + 5) = -3x$

d) $-24(10)x + 19x = 76x - (9x + 2)$

e) $8(x - 3) - 2(x - 2) = 20$

f) $x - 3 = 2(x + 5) + 2x + 2$

g) $\frac{1}{2}(x - 6) + \frac{3}{5}(x + 10) = 24$

h) $1 - \frac{x}{3} = 6$

i) $\frac{x}{2} - \frac{3x}{4} = 1$

j) $\frac{2x}{3} + \frac{7x}{6} = 5x + 30$

Answers:

a) x = 0

b) x = 1.6

c) x = 4

d) $x = \frac{1}{144}$

e) $x = 6\frac{2}{3}$

f) x = –5

g) $x = 19\frac{1}{11}$

h) x = –15

i) x = –4

j) $x = -9\frac{9}{19}$