Quadratic Formula

The quadratic formula can be used to find the roots of a quadratic equation of the form **$ax^2 + bx + c = 0$**. These roots correspond to the x-intercepts of the quadratic relation that the equation describes.

The quadratic formula is:

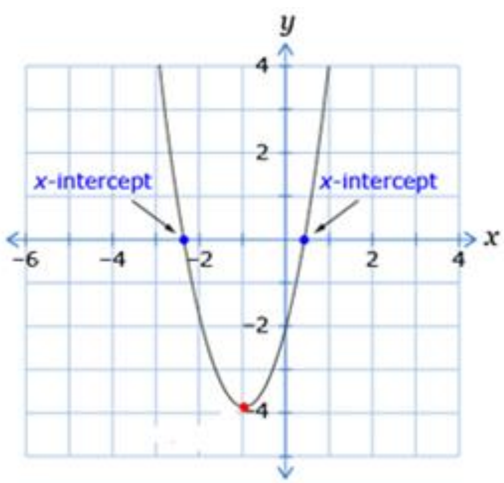
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

When working on solving quadratic equations, it is advisable to use the quadratic formula *only* when factoring fails.

The **$b^2 - 4ac$** part of the formula is called the **discriminant** and can be used to determine the number of roots or x-intercepts for a quadratic relation/equation.

Positive Discriminant

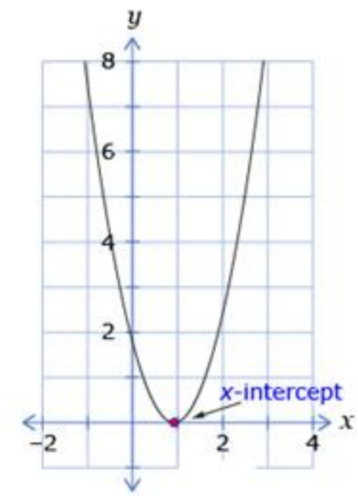
$b^2 - 4ac > 0$ (i.e. positive)



There are **2 real roots**, and 2 x-intercepts.

Zero Discriminant

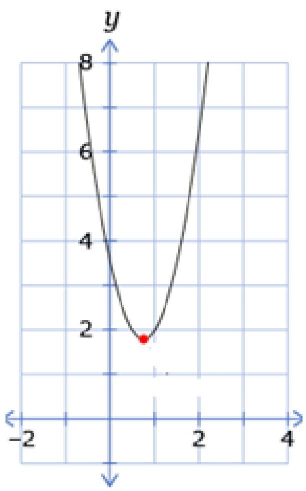
$b^2 - 4ac = 0$



There are **2 equal real roots**, and 1 x-intercept.

Negative Discriminant

$b^2 - 4ac < 0$ (i.e. negative)



There are **no real roots**, and no x-intercepts.

Example 1:

Use the quadratic formula to solve 5x + 19x – 21 = 0

Step 1:

Determine the values of a, b and c.

(Note: a is the number in front of the x^2 term, b is the number in front of the term, and c is the number on its own.)

Example:

From the equation, a = 5; b = 19 and c = –21.

Step 2:

Plug in the values for a, b and c into the quadratic formula.

Example:

$$x = \frac{-19 \pm \sqrt{19^2 - 4(5)(-21)}}{2(5)}$$

Step 3:

Simplify and solve.

Example:

$$x = \frac{-19 \pm \sqrt{781}}{10}$$

$$x = \frac{-19 \pm 27.94637722}{10}$$

$$x_1 = \frac{-19 + 27.946}{10} \quad x_2 = \frac{-19 - 27.946}{10}$$

$x_1 = 0.895$ $x_2 = -4.695$

Example 2:

Use the quadratic formula to find the x-intercept(s) of $y = 5x^2 - 3x + 9$, if any.

Step 1:

Determine the values of a, b, and c.

Example:

From the equation; $a = 5$, $b = -3$ and $c = 9$.

Step 2:

$$x = \frac{-(-3) \pm \sqrt{(-3)^2 - 4(5)(9)}}{2(5)}$$

Step 3:

Simplify and solve.

Example:

$$x = \frac{3 \pm \sqrt{-171}}{10}$$

Since we cannot take the square root of a negative number, the equation cannot be solved further using the set of real numbers. (Note: imaginary numbers are not discussed in this hand-out).

Step 4:

State the final answer.

Example:

Since there are no real solutions for this equation, there are no x-intercepts.

Example 3:

Use the quadratic formula to find the x-intercept(s) of $y = 4x^2 - 36x + 81$.

Step 1:

Determine the values of a, b and c.

Example:

From the equation; $a = 4$, $b = -36$ and $c = 81$.

Step 2:

Plug in the values for a, b and c into the quadratic formula.

Example:

$$x = \frac{-(-36) \pm \sqrt{(-36)^2 - 4(4)(81)}}{2(4)}$$

Step 3:

Simplify and solve.

Example:

$$x = \frac{36 \pm \sqrt{0}}{8}$$

x = 4.5

Step 4:

Therefore, there is one x-intercept which is (4.5, 0).

Practice Questions:

1) Solve using the quadratic formula. Round to 2 decimal places, if necessary.
   a) $0 = 4x^2 + 8x + 2$
   b) $0 = 2x^2 - 4x - 3$
   c) $9x^2 + 12x + 4 = 0$
   d) $3x^2 + 4x + 2 = 0$
   e) $-6x^2 + 18x = 1$
   f) $x(x-2) = 4$
2) Find the x-intercept(s) of each quadratic relation (if any) using the quadratic formula.
   a) $y = 2x^2 + 14x + 24.5$
   b) $y = x^2 + 1$
   c) $y = 7x^2 + 2x - 3$

Answers:

1a) $x_1 = -1.71$ $x_2 = -0.29$
1b) $x_1 = -0.58$ $x_2 = 2.58$
1c) $x = -0.67$
1d) no real solutions
1e) $x_1 = 0.06$ $x_2 = 2.94$
1f) $x_1 = -1.24$ $x_2 = 3.24$

2a) $(-3.5, 0)$
2b) no x-intercepts
2c) $(-0.81, 0)$ and $(0.53, 0)$

Where does the quadratic formula come from?

The quadratic formula can be derived by completing the square and isolating x in the standard form of a quadratic relation, $y = ax^2 + bx + c$, when $y = 0$ (since y is equal to 0 for any x-intercept).

Step 1:

Factor out "a" from $ax^2 + bx$

$$a(x^2 + \frac{b}{a}x) + c = 0$$

Step 2:

Add and subtract half of $\frac{b}{a}$ squared inside the brackets.

$$a\left[x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2\right] + c = 0$$

Step 3:

Factor the perfect square trinomial, $x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2$. Take $-\left(\frac{b}{2a}\right)^2$ out of the brackets by multiplying by "a."

$$a\left(x+\frac{b}{2a}\right)^2 - a\left(\frac{b}{2a}\right)^2 + c = 0$$

Step 4:

Simplify the $-a\left(\frac{b}{2a}\right)^2$ term and move to the right side of the equation. Move "c" to the right side of the equation.

$$a\left(x+\frac{b}{2a}\right)^2 = \frac{b2}{4a} - c$$

Step 5:

Move the "a" term to the right side of the equation by dividing $\frac{b^2}{4a}$ and "c" by "a"

$$\left(x+\frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

Step 6:

Subtract $\frac{b^2}{4a^2}$ and $\frac{c}{a}$ by finding a common denominator.

$$\left(x+\frac{b}{2a}\right)^2 = \frac{b^2-4ac}{4a^2}$$

Step 7:

Take the square root of both sides of the equation.

$$x+\frac{b}{2a} = \pm\sqrt{\frac{b^2-4ac}{4a^2}}$$

Step 8:

Simplify square root of $4a^2$ in the denominator on the right side of the equation. Move the $\frac{b}{2a}$ term to the right side.

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

Step 9:

Since both terms on the right side of the equation have a common denominator, add and subtract the two terms.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$