

HOSPITALITY Math Assessment Preparation Guide

Please note that the guide is for reference only and that it does not represent an exact match with the assessment content. The Assessment Centre at George Brown College is

Table of Contents

1. Operations with Whole Numbers	7
Addition of Whole Numbers (by hand):	7
Example:.....	7
Practice Questions:	7
Subtraction of Whole Numbers (by hand):	8
Example:.....	8
Practice Questions:	9
Multiplication of Whole Numbers (by hand):	9
Example.....	9
Practice Questions:	10
Division of Whole Numbers (by hand):	10
Example:.....	11
Practice Questions:	12
Order of Operations	12
Examples:	12
Practice Question:.....	13
2. Operations with Integers	13
Addition:	13
Example:.....	14
Practice question:	14
Example 1:.....	14
Example 2:.....	15
Practice questions:.....	15
Subtraction:	15
Examples:	15
Practice question:	15
Multiplication and Division	15
Examples:	16
Practice Questions:	16
3. Operations with Fractions	17
Converting a Mixed Number to an Improper Fraction	17
Example:.....	18
Practice Question:.....	18
Converting an Improper Fraction to a Mixed Number	18

Example:.....	19
Practice question:	19
Equivalent Fractions	19
Example:.....	20
Example:.....	21
Practice question:	21
Addition of Fractions	21
Examples:	22
Practice questions:.....	24
Subtraction of Fractions	24
Example:.....	24
Practice question:	25
Multiplication of Fractions.....	25
Examples:	25
Practice questions:.....	27
Examples:	27
Practice questions:.....	28
Ordering Fractions.....	29
Example:.....	29
Practice Question:.....	30
4. Operations with Decimals	31
Addition (by hand)	31
Practice questions:.....	32
Subtraction (by hand).....	32
Practice questions:.....	33
Multiplication (by hand)	33
Practice questions:.....	35
Division (by hand)	35
Example:.....	35
Example 2:.....	36
Practice questions:.....	36
5. Conversions between Fractions, Decimals and Percent.....	36
Converting Fractions to Decimals.....	36
Example:.....	37
Example:.....	37
Some useful fraction and decimal equivalents to remember:.....	37
Converting Decimals to Fractions.....	37
Example:.....	38
Practice question:	38
Introduction to Percent	38
Converting Decimals or Fractions to Percent	38

Example:.....	39
Example:.....	39
Practice question:	39
Converting Percent to Decimals or Fractions	39
Example:.....	40
Example:.....	40
Practice question:	40
6. Percent.....	41
Common Word problems involving percent:	41
Practice questions:	42
Sales Tax	42
Example:.....	43
Practice question:	43
Price Discounts.....	43
Example:.....	44
Practice question:	44
Percent Increase.....	44
Example:.....	45
Practice Question:.....	45
Percent Decrease	45
Example:.....	46
Practice Question:.....	46
Finding the Percent in Percent Increase or Percent Decrease.....	46
Examples:	47
Practice Questions:	47
Ratios and Proportion	48
Sample question:	48
Example:.....	48
Practice Questions:	49
8. Exponents.....	49
Examples:	50
Example:.....	50
Sample questions:	51
9. Scientific Notation	51
Examples:	52
Practice Questions:	53
Addition and Subtraction with Scientific Notation	53
Example:.....	53
Example 2:.....	54
Practice Questions:	55

Multiplication and Division with Scientific Notation	55
Example:.....	56
10. Square Roots	56
Practice question:	57
Example:.....	57
Practice Question:.....	57
Practice Question:.....	58
Solving One-variable Equations	58
11. Re-arranging Formulas	60
Example 1:.....	60
Example 2:.....	60
Sample question:	60
Example:.....	61
Sample question:	61
Definition of log:	62
Example:.....	62
Example 2:.....	62
Sample question:	63
Unit Conversions	63
Example:.....	64
Example:.....	64
Example:.....	64
Practice Questions:	65
Example:.....	66
Example 2:.....	67
Practice Questions:	67
Temperature Conversion Formulas	68
Example:.....	68
Example 2:.....	68
Practice Questions	68
13. Measures of Central Tendency: Mean, Median, Mode.....	68
Mean	68
Example:.....	69
Median	69
Example:.....	69
Mode.....	69
Example:.....	70
Practice Questions:	70
Appendix A: Glossary.....	71

Appendix B: Multiplication Table	74
Appendix C: Rounding Numbers	75
Example:.....	75
Example 2:.....	75
Example:.....	76
Appendix D: Answers to Practice Questions	77

Introduction

The Hospitality Math Assessment Preparation Guide is a reference tool for George Brown College students preparing to take the Hospitality math assessment. The study guide focuses on foundation-level math skills. **The study guide does not cover all topics on the assessment.**

The Hospitality Math Assessment Preparation Guide is organized around a select number of pre-algebra and arithmetics topics. It is recommended that users follow the guide in a step-by-step order as one topic builds on the previous one. Each section contains theory, examples with full solutions and practice question(s). Answers to practice questions can be found in Appendix D.

Reading comprehension and understanding of terminology are an important part of learning mathematics. Appendix A has a glossary of mathematical terms used throughout the Study Guide.

Subtraction of Whole Numbers (by hand):

To subtract two whole numbers by hand, first line up the numbers according to place values. Perform the subtraction one column at a time, starting at the right. If the number on the top is smaller than the number on the bottom, you can regroup 1 from the number in the next column to the left and add 10 to your smaller number.

Example:

Evaluate. $6012 - 189$.

thousands tens

6	0	1	2
-	1	8	9
	5	8	2

hundreds ones

Step 4:
Write 5 in the thousands place value.

Step 1:
 $2 - 9$ would give us a negative number. Thus, we need to regroup 1 from the tens place value; add 10 to 2 to get 12. $12 - 9 = 3$
 Write 3 in the ones place value.
 Note: In the tens place value, we have 0 left since we took 1 from 1.

Step 3:
 $9 - 1 = 8$
 Write 8 in the hundreds place value.

Step 2:
 $0 - 8$ would again give us a negative number. Thus, we need to regroup 1 from the hundreds place value, which is the next column to the left. The hundreds place value contains a 0, so we go to the next column to the left and regroup 1 from the thousands place value. This gives 10 in the hundreds place value and now we can borrow 1 from the hundreds, leaving 9 left over. Add 10 to 0 in the tens place values to get 10.
 $10 - 8 = 2$
 Write 2 in the tens place value.
 In the hundreds place value we have 9 left over and we have 5 left over in the thousands place value.

Final answer is 5823.

Practice Questions:

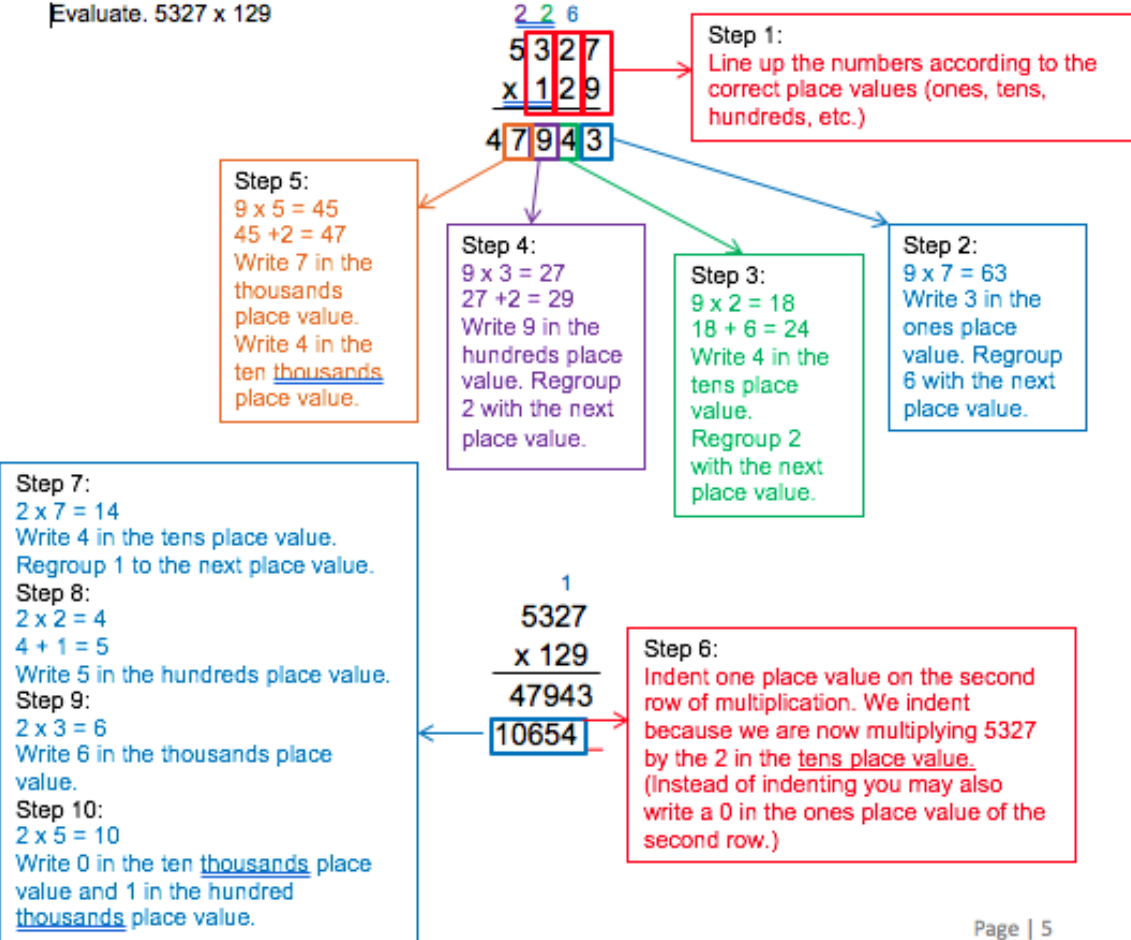
- Calculate $13567 - 2099$.
- Calculate $52001 - 1891$.

Multiplication of Whole Numbers (by hand):

To multiply two whole numbers by hand, first line up the numbers according to the last column. Next, multiply the number in the bottom row by each digit in the number in the top row, starting with the right-most digit (*Note: you MUST know the multiplication table for this step. A multiplication table can be found in Appendix B*). If the multiplication results in a two-digit number (i.e. a number greater than 9), regroup the tens place value with the next column by adding it to the product. Furthermore, every time you start a new row in your answer, starting with the second row, you need to indent one place value. Lastly, add all of the separate products to get the final answer.

Example

Evaluate. 5327×129



Step 1: Line up the numbers according to the correct place values (ones, tens, hundreds, etc.)

Step 2: $9 \times 7 = 63$
Write 3 in the ones place value. Regroup 6 with the next place value.

Step 3: $9 \times 2 = 18$
 $18 + 6 = 24$
Write 4 in the tens place value. Regroup 2 with the next place value.

Step 4: $9 \times 3 = 27$
 $27 + 2 = 29$
Write 9 in the hundreds place value. Regroup 2 with the next place value.

Step 5: $9 \times 5 = 45$
 $45 + 2 = 47$
Write 7 in the thousands place value. Write 4 in the ten thousands place value.

Step 6: Indent one place value on the second row of multiplication. We indent because we are now multiplying 5327 by the 2 in the tens place value. (Instead of indenting you may also write a 0 in the ones place value of the second row.)

Step 7: $2 \times 7 = 14$
Write 4 in the tens place value. Regroup 1 to the next place value.

Step 8: $2 \times 2 = 4$
 $4 + 1 = 5$
Write 5 in the hundreds place value.

Step 9: $2 \times 3 = 6$
Write 6 in the thousands place value.

Step 10: $2 \times 5 = 10$
Write 0 in the ten thousands place value and 1 in the hundred thousands place value.

Step 12:
 $1 \times 7 = 7$
 Write 7 in the hundreds place value.

Step 13:
 $1 \times 2 = 2$
 Write 2 in the thousands place value.

Step 14:
 $1 \times 3 = 3$
 Write 3 in the ten thousands place value.

Step 15:
 $1 \times 5 = 5$
 Write 5 in the hundred thousands place value.

$$\begin{array}{r} 5327 \\ \times 129 \\ \hline 47943 \\ 10654 \\ \hline \end{array}$$

Step 11:
 Indent two place values on the third row of multiplication. We indent because we are now multiplying 5327 by the 1 in the hundreds place value. (Instead of indenting you may also write two 0s in the ones and tens place values of the third row.)

$$\begin{array}{r} 5327 \\ \times 129 \\ \hline 12 \\ 47943 \\ + 10654 \\ 5327 \\ \hline 687183 \end{array}$$

Step 16:
 Add rows 1 to 3. Pay careful attention to adding column by column to make sure that you are adding the same place values together.

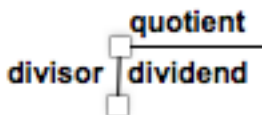
Final answer is 687183.

Practice Questions:

5. Evaluate. 2369×152 .
6. Evaluate. 1078×691 .

Division of Whole Numbers (by hand):

To divide two whole numbers by hand, set up the dividend and divisor as shown below. The quotient (the answer) will go on the line on top of the dividend.



Example:

Evaluate. $456 \div 3$.

$$\begin{array}{r} 3 \overline{)456} \end{array}$$

Step 1:
Set up the divisor and dividend as shown above. 456 is the dividend; 3 is the divisor.

$$\begin{array}{r} 1 \\ 3 \overline{)456} \\ \underline{-3} \\ 15 \end{array}$$

Step 2:
How many times does 3 go into 4 evenly? Once. Therefore, put 1 in the quotient.
 $3 \times 1 = 3$
Subtract 3 from 4.
1 remains.

$$\begin{array}{r} 15 \\ 3 \overline{)456} \\ \underline{-3} \\ 15 \end{array}$$

Step 3:
Bring the next number down, in this case 5. How many times does 3 go into 15 evenly? Five times. Therefore, put 5 in the quotient.
 $3 \times 5 = 15$
Subtract 15 from 15. Remainder is 0.

$$\begin{array}{r} 152 \\ 3 \overline{)456} \\ \underline{-3} \\ 15 \\ \underline{-15} \\ 06 \end{array}$$

Step 4:
Bring the next number, 6, down. How many times does 3 go into 6 evenly? Twice. Therefore, put 2 in the quotient.
 $3 \times 2 = 6$
Subtract 6 from 6. Remainder is 0 and there are no more numbers to divide 3 into. Thus, the division is done.

3

Final answer is 152.

Note: Division problems with two-digit divisors follow the same method as shown above.
Note: For division problems where the dividend does not divide evenly by the divisor, the quotient will have a remainder.

Practice Questions:

7. Calculate $12192 \div 8$.

8. Calculate $115620 \div 12$.

Order of Operations

As a *general* rule of thumb, the acronym BEDMAS can be followed for the correct order of operations.

Brackets

Exponents — Do in order from left to right.

Division }
Multiplication } Do in order from left to right.

Addition }
Subtraction } Do in order from left to right.

Important notes:

- If there are **multiple exponents**, evaluate the powers from left to right as they appear in the question.
- If there are **multiple brackets**, it does not matter which ones you do first, second, etc.
- If there are **multiple operations *within* brackets**, do the operations according to BEDMAS.
- **Division and multiplication** is done from left to right. This means that multiplication should be done before division, if it appears to the left of division.
- **Addition and subtraction** is done from left to right. Subtraction should be done first, if it is to the left of addition.
- For **rational expressions**, the numerator and denominator are evaluated separately according to BEDMAS. Then, determine the quotient.

Examples:

Evaluate.

a) $9 + (10 - 8)^2 \div 2 \times 3$ Step 1: Do the operation inside the brackets.

$= 9 + (2)^2 \div 2 \times 3$ Step 2: Evaluate the exponent.

$= 9 + 4 \div 2 \times 3$ Step 3: Do the division since it appears first going from left to right.

$$= 9 + 2 \times 3$$

Step 4: Do the multiplication.

$$= 9 + 6$$

Step 5: Do the addition.

$$= 15$$

Final answer is 15.

b)
$$\frac{2^3 + 13 \div 4}{9(3) - 2}$$

the bottom.

Step 1: Evaluate the exponent on the top. Do the multiplication on

$$= \frac{8 + 12 \div 4}{27 - 12}$$

Step 2: Do the division on the top.

$$= \frac{8 + 3}{27 - 12}$$

bottom.

Step 3: Do the addition on the top. Do the subtraction on the

$$= \frac{11}{15}$$

This is the final answer. The fraction cannot be reduced.

Practice Question:

9. Evaluate. $3 + 8 \times (9 - 2)^2$

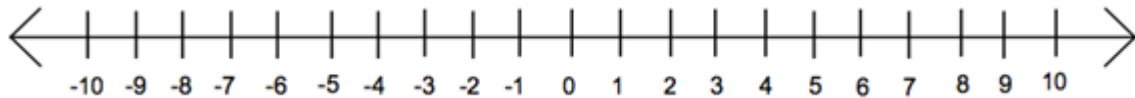
- a) 115
- b) 248
- c) 56
- d) 395

2. Operations with Integers

Integers refer to a set of numbers which is made up of positive whole numbers, zero and negative whole numbers. Integers do NOT include decimals or fractions.

Addition:

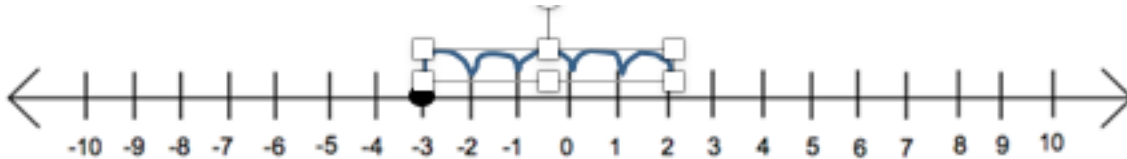
Addition of integers can be illustrated on a number line. Every integer has a position on a number line in relation to 0 and other integers. By convention, positive integers are written to the *right* of 0 and negative integers are written to the *left* of 0. Every space represents 1 on the number line below.



Example:

Find the sum of -3 and 5 .

Start at -3 and go 5 spaces to the right. The answer is 2 .



Practice question:

10. Use a number line to find the sum of -8 and 12 .

Drawing a number line can be time-consuming. The following “shortcut” rules may be used to add integers without a number line.

To add two integers with **different signs**, find the **difference** of the two numbers. The result has the sign of the bigger number.

To add two integers with **same signs**, find the **sum** of the two numbers. The result has the same sign as the two integers.

Note: By convention, an integer with no sign in front of it is positive.

Example 1:

Find the sum of -5 and 8 .

$$-5 + 8 = ?$$

-5 is negative and 8 is positive. The integers have different signs; thus, find the **difference** of the two numbers. The difference between 8 and 5 is 3 . Take the sign of $+8$ since 8 is bigger than 5 .

$$-5 + 8 = 3$$

Example 2:

Find the sum of -9 and -13 .

$$-9 + (-13) = ?$$

-9 and -13 are both negative. The integers have the **same** sign; thus, we find the sum of 9 and 13 which is 22. Keep the sign as negative.

$$-9 + (-13) = -22$$

Practice questions:

11. Find the sum of 15 and -22 .

12. Find the sum of -8 and -15 .

Subtraction:

To subtract two integers, change the question from subtraction to addition by adding the opposite of the given number.

Opposite refers to an integer with the opposite sign. Negative and positive are opposite signs.

Examples:

1. a) Evaluate. $-21 - 34$

$$\begin{aligned} -21 - 34 &= -21 + (-34) && [-34 \text{ is the opposite of } 34] \\ &= -45 \end{aligned}$$

b) Evaluate. $9 - (-45)$

$$\begin{aligned} 9 - (-45) &= 9 + (45) && [45 \text{ is the opposite of } -45] \\ &= 54 \end{aligned}$$

Practice question:

13. Evaluate. $21 - (-4)$

14. Evaluate. $-17 - 3$

Multiplication and Division

To **multiply or divide two integers**, perform the multiplication or division as you would with whole numbers. Add the correct sign to the answer according to the rules below.

$(+)(+) = +$	$(+) \div (+) = +$
$(+)(-) = -$	$(+) \div (-) = -$
$(-)(+) = -$	$(-) \div (+) = -$
$(-)(-) = +$	$(-) \div (-) = +$

One way to remember these rules is that **same signs** result in a **positive** and **different signs** result in a **negative**.

Examples:

- | | |
|------------------------|------------------------|
| 1. a) $(-3)(15) = -45$ | b) $21 \div (-7) = -3$ |
| c) $(-12)(-4) = 48$ | d) $-45 \div (-9) = 5$ |

Practice Questions:

15. Evaluate.
 $(-12)(-11) =$

16. Evaluate.
 $135 \div (-5) =$

17. Evaluate.
 $(-8)(3) + (-25)$

- a) 1
- b) -1
- c) -49
- d) 49

18. Evaluate.

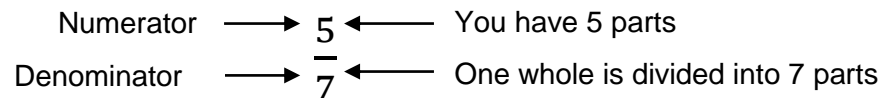
$$\frac{24}{-6} - (-6)(2)$$

- a) 8
- b) -8
- c) 4
- d) -20

3. Operations with Fractions

A **fraction** is a number that represents parts of a whole. The **numerator** represents the number of parts you have. The **denominator** is the number of parts into which one whole is divided.

Example:



An **improper fraction** is a fraction whose numerator is bigger than the denominator (e.g. $\frac{5}{3}$, $\frac{9}{2}$). This means that an improper fraction is *greater* than one whole.

A **mixed number** consists of a whole number and a fraction. (e.g. $2\frac{7}{8}$).

A mixed number can be written as an improper fraction and vice versa.

Converting a Mixed Number to an Improper Fraction

To convert a mixed number to an improper fraction:

- Multiply the whole number by the denominator and add to the numerator*.
- Keep the denominator the same**.

*This tells us how many parts we have in *total* (the parts that make up the whole number(s) and those in the numerator).

**We keep the denominator the same because we have NOT changed the number of parts one whole is divided into.

Example:

Convert $2\frac{7}{8}$ to an improper fraction.

$$2\frac{7}{8} = \frac{2(8)+7}{8} = \frac{23}{8}$$

Multiply whole number (2) by the denominator (8). Add the numerator (7).

Keep the denominator (8) the same

Practice Question:

19. Which of the following fractions is equivalent to $5\frac{7}{9}$?

- a) $\frac{35}{45}$
- b) $\frac{12}{9}$
- c) $\frac{52}{45}$
- d) $\frac{52}{9}$

Converting an Improper Fraction to a Mixed Number

To convert an improper fraction to a mixed number:

- Divide the numerator by the denominator. The answer should be a whole number and a remainder.
- The whole number is the whole number part of the mixed number.
- The remainder is the numerator of the fractional part of the mixed number.
- Keep the denominator the same.

Example:


Convert $\frac{9}{2}$ into a mixed number.

2 goes into 9 four times evenly. Thus, our mixed number will have 4 wholes and whatever fraction part is left over.

4 can be written as $\frac{8}{2}$.

$$\frac{9}{2} - \frac{8}{2} = \frac{1}{2}$$

Thus, we have $\frac{1}{2}$ left over as the fractional part of the mixed number.



$$\frac{9}{2} = 4\frac{1}{2}$$

Practice question:

20. What is $\frac{36}{7}$ written as a mixed number?

- a) $\frac{1}{7}$
- b) $5\frac{1}{7}$
- c) $6\frac{1}{7}$
- d) $5\frac{1}{5}$

Equivalent Fractions

Two fractions are considered equivalent if they are **equal in value**.

To determine if two given fractions are equivalent, **find the lowest terms of each fraction. If the lowest terms are the same, then the fractions are equivalent.** To find the lowest terms of a fraction, divide both the numerator and denominator by their greatest common factor.

Example:

$\frac{1}{2}$ is equivalent to $\frac{2}{4}$.

In the diagram below, you can see that $\frac{1}{2}$ and $\frac{2}{4}$ are equal in value. The large rectangle represents one whole. The first rectangle is divided into two equal parts (denominator = 2) and one part is coloured (numerator = 1). The second rectangle is divided into four equal parts (denominator = 4) and two parts are coloured (numerator = 2). The coloured parts occupy the same area in both diagrams because the fractions are equal in value.



However, $\frac{1}{2}$ is NOT equivalent to $\frac{2}{3}$.



As you can see in the diagram above, $\frac{2}{3}$ occupies more space than $\frac{1}{2}$. This is because $\frac{2}{3}$ is bigger than $\frac{1}{2}$ and the two fractions are NOT equivalent (i.e. not equal in value).

Also, notice that $\frac{2}{4}$ can be reduced to $\frac{1}{2}$ by dividing both the numerator and denominator of $\frac{2}{4}$ by 2. On the contrary, $\frac{2}{3}$ can NOT be reduced to $\frac{1}{2}$.

$$\frac{2 \div 2}{4 \div 2} = \frac{1}{2}$$

Therefore, $\frac{2}{4} = \frac{1}{2}$

But $\frac{1}{2} \neq \frac{2}{3}$

Note:

For any fraction, there is an infinite number of equivalent fractions.

Given a fraction, an equivalent fraction can be obtained by multiplying both the numerator and denominator by the same number.

Example:

Write two fractions that are equivalent to $\frac{2}{5}$.

$$\frac{2 \cdot 3}{5 \cdot 3} = \frac{6}{15}$$

$$\frac{2 \cdot 4}{5 \cdot 4} = \frac{8}{20}$$

Therefore, $\frac{6}{15}$ and $\frac{8}{20}$ are equivalent to $\frac{2}{5}$.

Note that both $\frac{6}{15}$ and $\frac{8}{20}$ can be reduced to $\frac{2}{5}$.

Practice question:

21. Which of the following fractions are equivalent to $\frac{7}{9}$?

- a) $\frac{14}{21}$
- b) $\frac{28}{35}$
- c) $\frac{21}{27}$
- d) none of the above

Addition of Fractions

- To add two fractions, they **MUST have the same (common) denominator**. Usually, it is best to find the *lowest* common denominator.
- The **lowest common denominator** is equal to the lowest common multiple of the two denominators. If one of the denominators is a multiple of another, then the lowest common multiple is the bigger denominator.
- Once you have a common denominator, change each fraction to an **equivalent fraction** with the desired denominator.
- Lastly, **add the numerators of the fractions** and **keep the denominator the same**.
- Reduce** the fraction to lowest terms and/or change an improper fraction to a mixed number, if necessary.

Examples:

1) What is the sum of $\frac{2}{3}$ and $\frac{5}{6}$?

$$\frac{2}{3} + \frac{5}{6} =$$

Step 1: Since the denominators of the fractions are different, we first need to find the lowest common denominator. In this case, the lowest common denominator is 6, since 6 is a multiple of 3.

$$\frac{2}{3} = \frac{2 \cdot 2}{3 \cdot 2} = \frac{4}{6}$$

Step 2: $\frac{2}{3}$ needs to be changed into an equivalent fraction with a denominator of 6. To do this, multiply the numerator, 2, and denominator, 3, by 2. The fraction, $\frac{5}{6}$, can stay as is, since it already has a denominator of 6.

$$\frac{4}{6} + \frac{5}{6} = \frac{4+5}{6} = \frac{9}{6}$$

Step 3: Add the numerators. Keep the denominator the same.

$$\frac{9}{6} = \frac{9 \div 3}{6 \div 3} = \frac{3}{2}$$

Step 4: $\frac{9}{6}$ can be reduced to $\frac{3}{2}$ by dividing both the numerator and denominator by 3.

$$\frac{3}{2} = 1\frac{1}{2}$$

Step 5: $\frac{3}{2}$ is an improper fraction and should be changed into a mixed number.

2) $\frac{3}{5} + \frac{7}{9} = ?$

First find the lowest common denominator. We are looking for the lowest common multiple of 5 and 9.

Note: If you are having trouble finding the lowest common multiple of two or more numbers, list the multiples for each number in a list. Look for the lowest multiple that appears in your lists.

Multiples of 5:

5, 10, 15, 20, 25, 30, 35, 40, **45**, 50, 55, 60, 65,

Multiples of 9:

9, 18, 27, 36, **45**, 54, 63, 72, ...

The lowest common multiple of 5 and 9 is 45.

In order to make $\frac{3}{5}$ into an equivalent fraction with a denominator of 45, multiply both the numerator and the denominator by 9. $\frac{3}{5} = \frac{3(9)}{5(9)} = \frac{27}{45}$

In order to make $\frac{7}{9}$ into an equivalent fraction with a denominator of 45, multiply both the numerator and the denominator by 5. $\frac{7}{9} = \frac{7(5)}{9(5)} = \frac{35}{45}$

$$\frac{27}{45} + \frac{35}{45} = \frac{62}{45}$$

$$\frac{62}{45} = 1\frac{17}{45}$$

3) $\frac{3}{16} + \frac{7}{12} = ?$

Find the lowest common denominator. We are looking for the lowest common multiple of 16 and 12.

Multiples of 16:

16, 32, **48**, 64, 80, 96,

Multiples of 12:

12, 24, 36, **48**, 60, 72, ...

The lowest common multiple of 16 and 12 is 48.

In order to make $\frac{3}{16}$ into an equivalent fraction with a denominator of 48, multiply both the numerator and the denominator by 3. $\frac{3}{16} = \frac{3(3)}{16(3)} = \frac{9}{48}$

In order to make $\frac{7}{12}$ into an equivalent fraction with a denominator of 48, multiply both the numerator and the denominator by 4. $\frac{7}{12} = \frac{7(4)}{12(4)} = \frac{28}{48}$

$$\frac{9}{48} + \frac{28}{48} = \frac{37}{48}$$

$\frac{37}{48}$ cannot be reduced.

Therefore, the sum is $\frac{37}{48}$.

Practice questions:

22. What is the sum of $\frac{2}{5}$ and $\frac{1}{15}$?

- a) $\frac{3}{20}$
- b) $\frac{7}{15}$
- c) $\frac{3}{15}$
- d) $\frac{7}{20}$

23. What is the sum of $\frac{1}{7}$ and $\frac{3}{5}$?

- a) $1\frac{1}{35}$
- b) 1
- c) $\frac{26}{35}$
- d) $\frac{4}{12}$

Subtraction of Fractions

To subtract two fractions, they **MUST have the same (common) denominator** (this is the same as adding fractions).

Once you have a common denominator, change each fraction to an *equivalent* fraction with the desired denominator.

Lastly, **subtract the numerators of the fractions** and **keep the denominator the same**. In subtraction, the order matters! Keep the order the same in how the question is written.

Reduce the fraction to lowest terms and/or change an improper fraction to a mixed number if necessary.

Example:

What is the difference between $\frac{13}{20}$ and $\frac{7}{50}$?

First, we need to find a common denominator for the two fractions. The lowest common multiple of 20 and 50 is 100.

$$\frac{13(5)}{20(5)} = \frac{65}{100}$$

$$\frac{7(2)}{50(2)} = \frac{14}{100}$$

Now, subtract the numerators and keep the denominator the same.

$$\frac{65}{100} - \frac{14}{100} = \frac{51}{100}$$

$\frac{51}{100}$ cannot be reduced.

The final answer is $\frac{51}{100}$.

Practice question:

24. What is the difference between $\frac{23}{63}$ and $\frac{2}{9}$?

- a) $\frac{21}{54}$
- b) $\frac{1}{7}$
- c) $\frac{21}{63}$
- d) $\frac{8}{63}$

Multiplication of Fractions

To multiply fractions:

- Multiply the numerator by the numerator
- Multiply the denominator by the denominator
- Reduce and/or change to a mixed number, if you can.

Note: a common denominator is NOT needed!

If **multiplying a mixed number by a fraction OR two mixed numbers**, first change the mixed number(s) to improper fraction(s) and then follow the same steps as above.

Examples:

1) What is the product of $\frac{13}{15}$ and $\frac{7}{5}$?

First, multiply the numerator by the numerator and denominator by the denominator.

$$\frac{13}{15} \times \frac{7}{5} = \frac{13(7)}{15(5)} = \frac{91}{75}$$

The product is an improper fraction. Change it to a mixed number.

$$\frac{91}{75} = 1\frac{16}{75}$$

Final answer is $1\frac{16}{75}$.

2) What is the product of $\frac{19}{20} \times \frac{4}{9}$?

First, multiply the numerator by the numerator and denominator by the denominator.

$$\frac{19(4)}{20(9)} = \frac{76}{180}$$

Now, reduce the product. 76 and 180 are both divisible by 4.

$$\frac{76 \div 4}{180 \div 4} = \frac{19}{45}$$

Note: It is also possible to reduce our fractions BEFORE we do the multiplication. This can be advantageous because the reducing happens with smaller numbers.

In our multiplication above, we can reduce 4 on the top and 20 on the bottom by dividing both numbers by 4. We can reduce a numerator from one fraction and denominator from another because order does not matter when multiplying numbers (i.e. 19×4 or 4×19 gives the same answer).

$$\div 4 = 1$$

$$\frac{19(\cancel{4})}{\cancel{20}(9)} = \frac{19(1)}{5(9)} = \frac{19}{45}$$

$$\div 4 = 5$$

3) What is the product of $2\frac{1}{9}$ and $\frac{3}{5}$?

Our first step is to change $2\frac{1}{9}$ into an improper fraction.

$$2\frac{1}{9} = \frac{19}{9}$$

Now, we can multiply numerator by the numerator and denominator by the denominator. We can also reduce before performing the actual multiplication (3 and 9 are both divisible by 3).

$$\div 3 = 1$$

$$\frac{19}{9} \times \frac{3}{5} = \frac{19(\cancel{3})}{\cancel{9}(5)} = \frac{19(1)}{3(5)} = \frac{19}{15}$$

$$\div 3 = 3$$

Since our product is an improper fraction, we need to change it to a mixed number.

$$\frac{19}{15} = 1\frac{4}{15}$$

The final answer is $1\frac{4}{15}$.

Practice questions:

25. What is the product of $\frac{4}{19}$ and $\frac{5}{2}$?

- a) $\frac{20}{36}$
- b) $\frac{20}{21}$
- c) $\frac{10}{19}$
- d) $\frac{8}{95}$

26. What is the product of $17\frac{1}{5}$ and $2\frac{2}{3}$?

- a) $45\frac{13}{15}$
- b) $40\frac{2}{15}$
- c) $11\frac{14}{30}$
- d) $34\frac{19}{20}$

Division of fractions

In order to divide two fractions:

- Change the division problem to a multiplication problem by **multiplying by the reciprocal of the divisor**. (To find the reciprocal of a fraction, switch the numerator and denominator).
- Multiply the fractions (see above).

If dividing a mixed number by a fraction OR two mixed numbers, first change the mixed number(s) to improper fraction(s) and then follow the same steps as above.

Examples:

1) What is the quotient of $\frac{19}{20}$ and $\frac{1}{5}$?

In order to divide $\frac{19}{20}$ and $\frac{1}{5}$, we need to multiply $\frac{19}{20}$ by the reciprocal of $\frac{1}{5}$.

The reciprocal of $\frac{1}{5}$ is $\frac{5}{1}$.

$$\frac{19}{20} \div \frac{1}{5} = \frac{19}{20} \times \frac{5}{1}$$

$$\begin{aligned} & \div 5 = 1 \\ \frac{19}{20} \times \frac{\cancel{5}}{1} &= \frac{19(1)}{4(1)} = \frac{19}{4} \\ & \div 5 = 4 \end{aligned}$$

The quotient is an improper fraction so we need to change it to a mixed number.

$$\frac{19}{4} = 4\frac{3}{4}$$

Final answer is $4\frac{3}{4}$.

2) What is the quotient of $3\frac{6}{7}$ and $\frac{9}{11}$?

First, we need to change $3\frac{6}{7}$ into an improper fraction.

$$3\frac{6}{7} = \frac{27}{7}$$

Now, we divide $\frac{27}{7}$ and $\frac{9}{11}$ by multiplying $\frac{27}{7}$ by the reciprocal of $\frac{9}{11}$.

$$\frac{27}{7} \div \frac{9}{11} = \frac{27}{7} \times \frac{11}{9}$$

$$\begin{aligned} & \div 9 = 3 \\ \frac{\cancel{27}}{7} \times \frac{11}{\cancel{9}} &= \frac{3(11)}{7(1)} = \frac{33}{7} \\ & \div 9 = 1 \end{aligned}$$

Last step is to change $\frac{33}{7}$ to a mixed number.

$$\frac{33}{7} = 4\frac{5}{7}$$

Practice questions:

27. What is the quotient of $\frac{5}{11}$ and $\frac{4}{33}$?

- a) $\frac{15}{4}$
- b) $\frac{20}{363}$
- c) $3\frac{3}{4}$
- d) Both a and c

28. What is the quotient of $\frac{73}{90}$ and $3\frac{3}{5}$?

- a) $\frac{73}{324}$
- b) $\frac{219}{150}$
- c) $\frac{1314}{150}$
- d) $\frac{324}{73}$

Ordering Fractions

There are a numbers of ways to compare (greater than, less than, equal to) or order fractions.

One way is to convert all fractions to equivalent fractions with a common denominator and then to compare the numerators. Given the same denominator, a fraction with a bigger numerator will be greater than a fraction with a smaller numerator.

Another approach is to **convert all fractions to decimals and then use the decimals to compare or order the given fractions.** (See section on converting fractions to decimals.)

Example:

Order the following numbers from least to greatest.

$$\frac{2}{3}, \frac{5}{6}, \frac{7}{12}, 2, 1\frac{1}{2}$$

Method 1: Using a common denominator

The lowest common multiple of 3, 6, 12, 1 and 2 is 12. Thus, the lowest common denominator for the fractions is 12.

$$\frac{2}{3} = \frac{8}{12}$$

$$\frac{5}{6} = \frac{10}{12}$$

$$2 = \frac{2}{1} = \frac{24}{12}$$

$$1\frac{1}{2} = \frac{3}{2} = \frac{18}{12}$$

$$\frac{7}{12} < \frac{8}{12} < \frac{10}{12} < \frac{18}{12} < \frac{24}{12}$$

Therefore,

$$\frac{7}{12} < \frac{2}{3} < \frac{5}{6} < 1\frac{1}{2} < 2$$

Method 2: Converting fractions to decimals

$$\frac{2}{3} = 0.66666667$$

$$\frac{5}{6} = 0.83333333$$

$$\frac{7}{12} = 0.58333333$$

$$1\frac{1}{2} = 1.5$$

$$0.58333333 < 0.66666667 < 0.83333333 < 1.5 < 2$$

Therefore,

$$\frac{7}{12} < \frac{2}{3} < \frac{5}{6} < 1\frac{1}{2} < 2$$

Practice Question:

29. Which of the following numbers are greater than $\frac{7}{15}$?

i) $\frac{1}{2}$

ii) $\frac{1}{3}$

iii) $1\frac{1}{4}$

iv) $\frac{9}{10}$

a) i and ii

b) i, ii and iii

c) ii and iv

d) i, iii and iv

4. Operations with Decimals

Addition (by hand)

To add two decimals by hand, first line up the decimals according to place values. The decimal points should be lined up one on top of another. Perform the addition one column at a time starting at the right. The number in the tens place value of any two-digit numbers must be regrouped with the next column. The decimal point in the answer goes directly underneath the decimal points in the numbers being added. This is why it is so **important to line up the decimals correctly!**

Example: $96.45 + 3.987$

$$\begin{array}{r}
 96.45 \\
 + 3.987 \\
 \hline
 \end{array}$$

Make sure to line up the place values according to the decimal point. The decimal point will go in the SAME position in your answer.

$9 + 1 = 10$ Write 0. Regroup 1 to the left.	$6 + 3 + 1 = 10$ Write 0. Regroup 1 to the left.	$4 + 9 + 1 = 14$ Write 4. Regroup 1 to the left.	$5 + 8 = 13$ Write 3. Regroup 1 to the left.	$7 + 0 = 7$	If you wish, you may write 0 to fill in the empty place value.
$1 \ 0 \ 0 \ . \ 4 \ 3 \ 7$	$9 \ 6 \ . \ 4 \ 5 \ 0$	$+ \ 3 \ . \ 9 \ 8 \ 7$	\hline		

Practice questions:

30. Add $35.879 + 1.36$

31. Add $0.0369 + 1.099$

Subtraction (by hand)

To subtract two decimals by hand, first line up the decimals according to place values. The decimal points should be lined up one on top of another. Perform the subtraction one column at a time starting at the right. If the number on the top is smaller than the number on the bottom, you can regroup 1 from the number in the next column to the left and add 10 to your smaller number. The decimal point in the answer goes directly beneath the decimal points in the numbers being subtracted. Any “empty” decimal place values can be filled in with 0’s.

Example: $16.9 - 7.804$

$$\begin{array}{r}
 16.9 \\
 - 7.804 \\
 \hline
 \end{array}$$



Make sure to line up the place values according to the decimal point. The decimal point will go in the SAME position in your answer

$$\begin{array}{r}
 0 \quad 8 \quad 9 \\
 16.900 \\
 - 7.804 \\
 \hline
 9.096
 \end{array}$$

Step 1: Write 0 in the empty place values.

Step 2: 0 minus 4 gives a negative answer. Thus, regroup 1 from the next column to the left. There is a 0 in the next column. Thus, regroup 1 from the next column, which has 9. 1 is regrouped with the rightmost 0 to make ten. 8 and 9 is left in the next two columns to the left. $10 - 4 = 6$. Write 6 directly below 0 and 4.

Step 4: $8 - 8 = 0$. Write 0 directly

Step 3: $9 - 0 = 9$. Write 9 directly below.

Step 5: $6 - 7$ gives a negative answer. Thus, regroup 1 from the next column to the left. 0 is left in this column. $16 - 7 = 9$. Write 9 directly below.

Practice questions:

32. Subtract 369.8 – 1.539

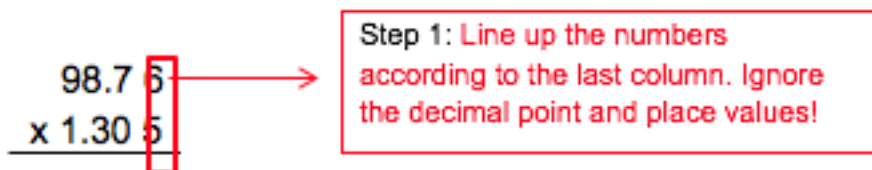
33. Subtract 10.003 – 1.678

Multiplication (by hand)

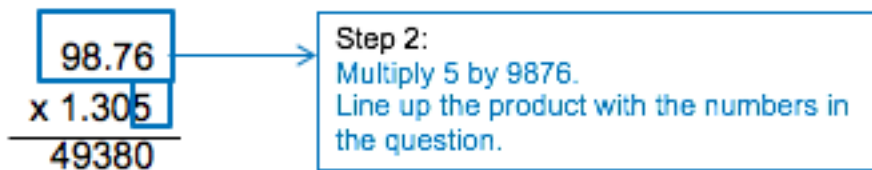
To multiply two decimals by hand, first line up the numbers according to the last column. **Ignore the decimal points for now.** Next, follow the same procedure as multiplying whole numbers. Lastly, count the total number of decimal places between the two decimals. Starting at the right most decimal place in the answer, count the same number of decimal places going left. Place the decimal point here. The number of decimal places in the answer should be the same as the total number of decimal places in the question.

Example:

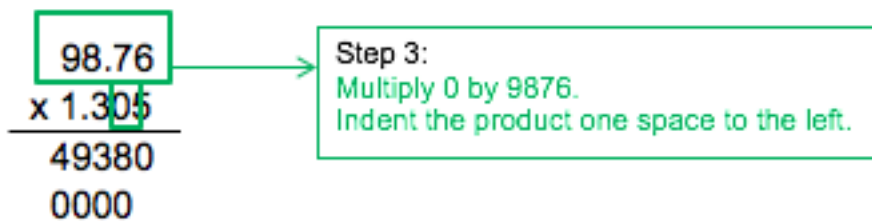
Evaluate. 98.76×1.305 .



Step 1: Line up the numbers according to the last column. Ignore the decimal point and place values!

$$\begin{array}{r} 98.76 \\ \times 1.305 \\ \hline \end{array}$$


Step 2: Multiply 5 by 9876. Line up the product with the numbers in the question.

$$\begin{array}{r} 98.76 \\ \times 1.305 \\ \hline 49380 \end{array}$$


Step 3: Multiply 0 by 9876. Indent the product one space to the left.

$$\begin{array}{r} 98.76 \\ \times 1.305 \\ \hline 49380 \\ 0000 \end{array}$$

$\begin{array}{r} 98.76 \\ \times 1.305 \\ \hline 49380 \\ 0000 \\ 29628 \end{array}$	<p>Step 4: Multiply 3 by 9876. Indent the product two spaces to the left.</p>
$\begin{array}{r} 98.76 \\ \times 1.305 \\ \hline 49380 \\ 0000 \\ 29628 \end{array}$	<p>Step 5: Multiply 1 by 9876. Indent the product three spaces to the left.</p>

$\begin{array}{r} 98.76 \\ \times 1.305 \\ \hline 49380 \\ + 0000 \\ 29628 \\ 9876 \\ \hline 12888180 \end{array}$	<p>Step 6: Add the products column by column starting at the right.</p>
--	---

$\begin{array}{r} 98.76 \\ \times 1.305 \\ \hline 49380 \\ + 0000 \\ 29628 \\ 9876 \\ \hline 128.88180 \end{array}$	<p>Step 7: Count the <i>total</i> number of decimal places in the numbers being multiplied, five in this example. Your answer will have the <i>same</i> number of decimal places. Starting at the right-most digit, count the appropriate number of spaces and place the decimal point.</p>
---	--

The final answer is 128.8818. The last 0 can be dropped as this does not change the *value* (or magnitude) of the number.

Practice questions:

34. Multiply 12.89 by 3.671

35. Multiply 0.0031 by 127.9

Division (by hand)

One way to divide two decimals is to make the divisor a whole number. To do this, count the number of decimal places in the divisor and multiply by a multiple of 10 that will make the divisor a whole number (Rule of thumb: the number of zeros in the multiple of 10 should equal the number of decimal places you are trying to get rid of). For example, 12.4 should be multiplied by 10 since multiplying by 10 moves the decimal place one place to the right. $12.4 \times 10 = 124$. However, in order to keep the division problem equivalent to the question given, you **MUST** also multiply the dividend by the same multiple of 10! Note that, the dividend can remain a decimal. For example, to divide 62.08 by 3.2, multiply the dividend and divisor by 10. The division problem becomes $620.8 \div 32$. Once the divisor is a whole number, proceed to do the division problem as you would divide any two whole numbers. If the dividend is a decimal, put a decimal point in the quotient once you “meet” the decimal point in the dividend during long division (see Example 2).

Example:

Evaluate. $358.8 \div 1.2$

$1.2 \times 10 = 12$

Step 1: You want to make the divisor, 1.2, into a whole number. Since 1.2 has one decimal place, multiply by 10.

$358.8 \times 10 = 3588$

Step 2: Since 1.2 was multiplied by 10, the dividend, 358.8 **MUST** also be multiplied by 10.

The division problem becomes $3588 \div 12$.

$$\begin{array}{r}
 299 \\
 12 \overline{) 3588} \\
 \underline{-24} \\
 118 \\
 \underline{108} \\
 108 \\
 \underline{108} \\
 0
 \end{array}$$

Step 3: Do long division as you would normally divide two whole numbers.

Final answer is 299.

Example 2:

Evaluate. $21.6279 \div 0.07$.

$0.07 \times 100 = 7$

Step 1: You want to make the divisor, 0.07, into a whole number. Since 0.07 has two decimal places, multiply by 100.

$21.6279 \times 100 = 2162.79$

Step 2: Since 0.07 was multiplied by 100, the dividend, 21.6279 MUST also be multiplied by 100.

The division problem becomes $2162.79 \div 7$.

$$\begin{array}{r}
 308.97 \\
 7 \overline{) 2162.79} \\
 \underline{21} \\
 062 \\
 \underline{56} \\
 67 \\
 \underline{63} \\
 49 \\
 \underline{49} \\
 0
 \end{array}$$

Step 3: Do long division as you would normally divide two whole numbers. Once you “meet” the decimal point in the dividend, put a decimal point in the quotient.

Put a decimal point in the quotient before bringing the 7 down.

Practice questions:

36. Evaluate. $20.79 \div 1.1$

37. Evaluate. $0.08265 \div 0.05$

5. Conversions between Fractions, Decimals and Percent

Converting Fractions to Decimals

To convert a proper or improper fraction into a decimal, divide the numerator by the denominator using long division or a calculator.

The decimal equivalent of a proper fraction will always be less than 1 but greater than 0. The decimal equivalent of an improper fraction will always be greater than 1.

Example:

Convert $\frac{6}{15}$ into a decimal.

$$6 \div 15 = 0.4$$

To convert a mixed number into a decimal, divide the numerator by the denominator for the fractional part and then add the whole number to the resulting decimal.

Example:

Convert $3\frac{1}{8}$ into a decimal.

$$1 \div 8 = 0.125$$

$$0.125 + 3 = 3.125$$

Some useful fraction and decimal equivalents to remember:

$\frac{1}{2} = 0.5$	$\frac{1}{5} = 0.2$
$\frac{1}{3} = 0.333\dots$	$\frac{2}{5} = 0.4$
$\frac{2}{3} = 0.666\dots$	$\frac{3}{5} = 0.6$
$\frac{1}{4} = 0.25$	$\frac{4}{5} = 0.8$
$\frac{3}{4} = 0.75$	$\frac{1}{10} = 0.1$

Converting Decimals to Fractions

To convert a decimal to a fraction, follow the following steps:

- Write the decimal in fraction form with a denominator of 1 and decimal in the numerator
- Multiply both numerator and denominator by a multiple of 10 that will make the decimal a whole number.
- Reduce the fraction if you can.

Example:

Convert 0.82 into a fraction.

First, write the decimal in fraction form by writing the decimal over 1.

$$\frac{0.82}{1}$$

Next, multiply both the numerator and denominator by 100 since we want to get rid of two decimal places in 0.82.

$$\frac{0.82}{1} \times \frac{100}{100} = \frac{82}{100}$$

Now, reduce the fraction.

$$\frac{82}{100} = \frac{41}{50}$$

The final answer is $\frac{41}{50}$.

Practice question:

38. Which of the following fractions are equivalent to 0.05?

- a) $\frac{19}{20}$
- b) $\frac{1}{20}$
- c) $\frac{1}{2}$
- d) $\frac{1}{10}$

Introduction to Percent

Percent means “per *hundred*”.

Like fractions, percent are a way to represent parts of one whole. However, in percent one whole is always considered to be 100%.

1 whole = 100%

Converting Decimals or Fractions to Percent

To convert a fraction into a percent, multiply by $\frac{100}{1}$ and then simplify as much as you can (e.g. reduce the fraction, convert an improper fraction to a mixed number, etc.).

Another way to convert a fraction into a percent is to convert the fraction into a decimal first and then multiply by 100.

Example:

Convert $\frac{1}{3}$ into a percentage.

$$\frac{1}{3} \times \frac{100}{1} = \frac{100}{3}$$

Step 1: Multiply the fraction by $\frac{100}{1}$.

$$\frac{100}{3} = 33 \frac{1}{3}$$

Step 2: Change the improper fraction into a mixed number.

Final answer is $33 \frac{1}{3} \%$.

To convert a decimal into a percent, multiply the decimal by 100. The shortcut method with multiplying decimals by 100 is to move the decimal place over to the right two times.

Example:

Convert 0.028 into a percentage.

$$0.028 \times 100 = 2.8\%$$

Practice question:

39. What is $\frac{2}{5}$ expressed as a percent?

- a) $\frac{2}{5} \%$
- b) 0.4 %
- c) 4%
- d) 40%

Converting Percent to Decimals or Fractions

To convert a percent into a fraction, write the percent as the numerator of a fraction with a denominator of 100. (This is the same as dividing the percent by 100). Simplify fully.

$$\frac{\textit{percent}}{100}$$

Example:

Convert 52% into a fraction.

$$52\% = \frac{52}{100}$$

Step 1: Write the percent as the numerator. Denominator is 100.

$$\frac{52}{100} = \frac{13}{25}$$

Step 2: Reduce the fraction.

Final answer is $\frac{13}{25}$.

To convert a percent into a decimal, divide the percent by 100. The shortcut method to divide decimals by 100 is to move the decimal place over to the left two times.

Example:

Convert 3.9% into a decimal.

$$3.9\% \div 100 = 0.039$$

Final answer is 0.039.

Practice question:

40. What is 34% expressed as a fraction?

- a) $\frac{34}{50}$
- b) $\frac{34}{1}$
- c) $\frac{17}{50}$
- d) $\frac{17}{100}$

6. Percent

Common Word problems involving percent:

Examples of the three most common word problems involving percent are:

- What is 15% of 30?
- 30 is 15% of what number?
- 15 is what percent of 30?

To solve word problems in the format “What is x% of y?”, follow these steps:

- 1. Convert the percentage into a decimal.**
- 2. Multiply the decimal by the number given in the question.**

Solution for “What is 15% of 30?”

$$15\% = 15 \div 100 = 0.15$$

$$0.15 \times 30 = 4.5$$

Therefore, 15% of 30 is 4.5.

To solve word problems in the format “y is x% of what number?”, follow these steps:

- 1. Convert the percentage into a decimal.**
- 2. Divide the number given in the question by the decimal.**

Solution for “30 is 15% of what number?”

$$15\% = 15 \div 100 = 0.15$$

$$30 \div 0.15 = 200$$

Therefore, 30 is 15% of 200.

To solve word problems in the format “x is what percent of y?”, follow these steps:

- 1. Divide the part, x, by the whole, y.**
- 2. Multiply the resulting quotient by 100.**

Solution for “15 is what percent of 30?”

$$15 \div 30 = 0.5$$

$$0.5 \times 100 = 50$$

Therefore, 15 is 50% of 30.

Practice questions:

41. What is 60% of 500?

- a) 3000
- b) 300
- c) 30
- d) 833.33

42. 15 is 75% of what number?

- a) 20
- b) 200
- c) 11.25
- d) 5

43. 200 is what percent of 50?

- a) 40%
- b) 25%
- c) 4%
- d) 400%

Sales Tax

Sales tax is expressed as a certain percentage of a sales price. The sales tax amount (\$) is *added* to the selling price in order to get the final total price.

When solving for the total price of a product after taxes use the following steps:

Method 1:

1. Find the sales tax amount by converting the sales tax percentage into a decimal and then multiplying by the selling price.
2. Add the sales tax amount to the original selling price.

OR

Method 2:

1. Convert the sales tax percentage into a decimal. Then add 1. (The decimal part of this number represents the amount of tax; 1 represents the original sales price).
2. Multiply the number from the previous step by the original sales price to get the total final price.

Note: when working with dollar amounts, always round your answer to 2 decimal places.

Example:

A t-shirt costs \$20.99. What is the final price of this t-shirt after a tax of 13% is added?

Method 1:

Step 1: $13\% \div 100 = 0.13$ $0.13 \times 20.99 = 2.73$

Step 2: $20.99 + 2.73 = 23.72$

Therefore, the final price of the t-shirt is \$23.72.

Method 2:

Step 1: $13\% \div 100 = 0.13$

$0.13 + 1 = 1.13$

Step 2: $1.13 \times 20.99 = 23.72$

Therefore, the final price of the t-shirt is \$23.72.

Practice question:

44. What is the final price of a textbook that costs \$187 after 13% tax is added?

- a) 211.31
- b) 199.99
- c) 24.31
- d) 238.46

Price Discounts

Discount is usually expressed as a certain percentage of a selling price (e.g. 30% off). The sales tax amount (\$) is *subtracted* from the selling price in order to get the final price.

When solving for the total price of a product after a discount has been applied use the following steps:

Method 1:

1. Find the discount amount by converting the discount percentage into a decimal and then multiplying by the original price.
2. Subtract the discount amount from the original price.

OR

Method 2:

1. Convert the discount percentage into a decimal. Subtract this decimal from 1. (This decimal represents what *part* of the original price is the discounted price.)
2. Multiply the number from the previous step by the original price to get the final discounted price.

Example:

A camera costs \$299. Right now, the camera is on sale at 15% off. What is the discounted price of the camera?

Method 1:

Step 1: $15\% \div 100 = 0.15$

$$0.15 \times 299 = 44.85$$

Step 2: $299 - 44.85 = 254.15$

Therefore, the discounted price of the camera is \$254.15.

Method 2:

Step 1: $15\% \div 100 = 0.15$

$$1 - 0.15 = 0.85$$

Step 2: $0.85 \times 299 = 254.15$

Therefore, the discounted price of the camera is \$254.15.

Practice question:

45. A box of pens costs \$17.99. The pens are currently on sale at 30% off. What is the discounted price of a box of pens?

- a) 14.99
- b) 59.97
- c) 5.40
- d) 12.59

Percent Increase

Questions that involve a **percent increase** can be solved using the same strategies as for prices after taxes. Taxes are really just a percent increase.

To find the number **after percent increase**:

Step 1. Express percent as a decimal by dividing the percent by 100.

Step 2. Multiply the decimal by the number representing the total to get the increase amount.

Step 3. Add the increase amount and the original amount.

Example:

The sales of company A totaled \$389,000 in 2010. In 2011, the sales increased by 3%. What is the amount of sales for company A in 2011?

Solution:

$$3\% \div 100 = 0.03$$

$$0.03 \times 389000 = 11670$$

$$389000 + 11670 = 400670$$

Therefore, the sales in 2011 were \$400, 670.

Practice Question:

46. In the fall semester, a total of 780 students visited the Tutoring and Learning Centre. In the winter semester, the number of students increased by 5%. What is the number of students who visited the Tutoring and Learning Centre in the winter semester?

- a) 15600
- b) 1170
- c) 939
- d) 819

Percent Decrease

Questions that involve a **percent decrease** can be solved using the same strategies as for prices after discount. Discounts are really just a percent decrease.

To find the number **after percent decrease**:

Step 1. Express percent as a decimal by dividing the percent by 100.

Step 2. Multiply the decimal by the number representing the total to get the decrease amount.

Step 3. Subtract the decrease amount from the original amount.

Example:

The population of a small northern town was 6780 people. This year, the population decreased by 3.2%. What is the population of the town this year?

Solution:

$$3.2\% \div 100 = 0.032$$

$0.032 \times 6780 = 216.96 = 217$ (We cannot have 0.96 of a person. Therefore, round the answer to the nearest whole number.)

$$6780 - 217 = 6563$$

Therefore, the population of the town this year is 6563 people.

Practice Question:

47. Last year there were 3180 cases of the flu treated at hospital A. This year, the number of cases of flu decreased by 12.7%. What is the number of cases of flu for this year?

- a) 404
- b) 2776
- c) 3584
- d) 3139

Finding the Percent in Percent Increase or Percent Decrease.

When finding the percent by which a number has increased or decreased, use the following guidelines:

Find the difference between the original number and the number after increase/decrease

1. Divide the difference by the original number. (You should get a decimal answer.)
2. Multiply the decimal by 100 to get a percentage.

Note: The percent can be larger than 100%.

Examples:

Problem 1:

The average price of a detached home in city A was \$506,000 in 2010. By 2012, the average price increased to \$745,000. By what percentage has the average home price increased from 2010 to 2012? (State the final answer to 2 decimal places.)

Solution:

$$745000 - 506000 = 239000$$

$$239000 \div 506000 = 0.4723$$

$$0.4723 \times 100 = 47.23\%$$

Therefore, the average home price increased by 47.23% from 2010 to 2012.

Problem 2:

In 1990 the number of families in the GTA relying on social assistance was 35700. By 2000 this number decreased to 32100. By what percentage did the number of families in the GTA relying on social assistance decrease between 1990 and 2000? (State the final answer to the nearest whole percent.)

Solution:

$$35700 - 32100 = 3600$$

$$3600 \div 35700 = 0.10$$

$$0.10 \times 100 = 10\%$$

Therefore, there was a 10% decrease in the number of families in the GTA relying on social assistance between 1990 and 2000.

Practice Questions:

48. Due to inflation, the price of food has increased. If a loaf of bread cost \$1.99 five years ago and costs \$2.49 today, what is the percent increase in the price of a loaf of bread?

- a) 20%
- b) 25%
- c) 10%
- d) 80%

49. A company's profit was \$897,000 in 2010. Following recession, the company's profit decreased to \$546,000 in 2011. What was the percent decrease in the company's profit between 2010 and 2011?
- a) 64%
 - b) 27%
 - c) 13%
 - d) 39%

Ratios and Proportion

A **ratio** is a comparison of one number to another.

The order, in which the ratio is written, matters.

For example, 1 (apple) to 2 (oranges) is not the same as 2 (apples) to 1 (orange).

Ratios can be written in three different ways:

With a colon
e.g. 5:3

In words
e.g. 5 to 3

As a fraction
e.g. $\frac{5}{3}$

Ratios should be simplified to **lowest terms** (like fractions).

For example, 6:3 can be simplified to 2:1 by dividing *each* number in the ratio by 3.

Sample question:

50. In a bag there are 7 red marbles for every 21 green marbles. Write the ratio of green to red marbles in **lowest terms**.

A **proportion** is made up of two **equal ratios**.

For example, 2:3 = 4:6

Proportions can be solved using *cross multiplication*.

Example:

Solve the following proportion.

$$3:7 = 45:x$$

$$\frac{3}{7} = \frac{45}{x}$$

Step 1: Write the proportion in fraction form. Pay special attention to the *order* of numbers.

$$\frac{3}{7} \times \frac{45}{x}$$

Step 2: Cross multiply. Remember to keep the equal sign between the two sides.

$$3x = 45(7)$$

Step 3: Move 3 to the right side of the equation by dividing the right side by 3.

$$x = \frac{45(7)}{3}$$

Step 4: Reduce the 45 and 3 by dividing each number by 3.

$$x = \frac{15(7)}{1}$$

Step 5: Evaluate the right side of the equation.

$$x = 105$$

Final answer.

Practice Questions:

51. Solve for x.

$$2:3 = x:18$$

52. Solve for x.

$$x:2 = 3:4$$

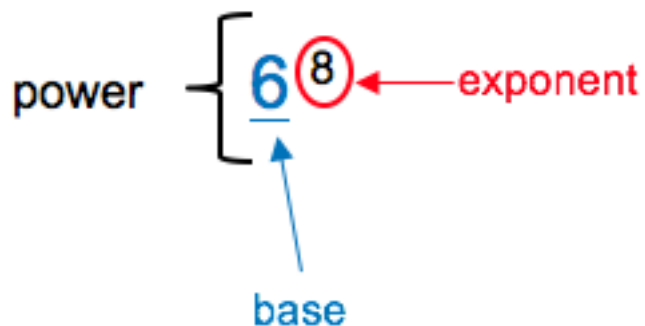
53. Solve for x.

$$\frac{x}{12} = \frac{10}{60}$$

54. Solve for x.

$$\frac{3}{x} = \frac{30}{5}$$

8. Exponents



Exponents are used to write **repeated multiplication**.

6^8 written in expanded form as repeated multiplication is $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6$.

When working with powers, it is very **important to identify the base correctly**.

If the base on a power is negative, it must be included in brackets.

For example, $(-8)^2 = (-8)(-8) = 64$

If there are *no* brackets, the base is positive but the power is negative.

For example, $-8^2 = -(8)(8) = -64$

As you can see from the above example, $(-8)^2$ is NOT the same as -8^2 .

Examples:

Write each power in expanded form as repeated multiplication.

$$-3^4 = - (3)(3)(3)(3)$$

$$(-9)^3 = (-9)(-9)(-9)$$

$$-(-2)^5 = -(-2)(-2)(-2)(-2)(-2)$$

To **evaluate a power** means to find its numerical value. This can be done on a calculator or by hand using repeated multiplication. Be extra careful with negative signs!

Example:

Evaluate the following powers.

- a) -7^3
- b) 9^4
- c) $(-2)^5$
- d) $(\frac{3}{4})^2$

Solutions:

- a) $-7^3 = -7 \times 7 \times 7 = -343$
- b) $9^4 = 9 \times 9 \times 9 \times 9 = 6561$
- c) $(-2)^5 = (-2) (-2) (-2) (-2) (-2) = -32$
- d) $\left(\frac{3}{4}\right)^2 = \left(\frac{3}{4}\right) \left(\frac{3}{4}\right) = \frac{9}{16}$

Sample questions:

55. What is -15^3 written as repeated multiplication?

- a) $(-15)(-15)(-15)$
- b) $(-15)(15)(-15)$
- c) $(15)(15)(15)$
- d) $(-15)(15)(15)$

56. Evaluate $(-7)^3$

- a) -49
- b) 49
- c) 343
- d) -343

9. Scientific Notation

Scientific notation is commonly used to represent very large or very small numbers in a convenient way.

Scientific notation is written in the following format using a power with base 10.

$m \times 10^b$

m represents the mantissa.

$1 \leq m < 10$

The mantissa MUST be equal to or greater than 1 but less than 10.

b is the exponent on a power with base 10.

Very large numbers will have a positive exponent.

Very small numbers will have a negative exponent.

To write a number in scientific notation:

- Move the decimal to a position immediately to the right of the first nonzero digit. Scan the number from left to right.
- Count the number of place values you had to move the decimal point. This is the value of the exponent.
 - If you moved the decimal point to the **left**, make the **exponent positive**.
 - If you moved the decimal point to the **right**, make the **exponent negative**.
- Drop all trailing or leading zeroes. The remaining number is the mantissa.
- Write the number in scientific notation as a product of the mantissa and power with base 10.

Examples:

1. Write 450,900,000,000 in scientific notation.

4.50,900,000,000 **Step 1: Move the decimal to a position immediately to the right of the first nonzero digit. Scan the number from left to right.**

4.50,900,000,000 **Step 2: Count the number of place values the decimal point was moved. In this case, the decimal point was moved 11 spaces to the left. Therefore, the exponent on base 10 will be positive 11.**

4.50,9~~00,000,000~~ **Step 3: Drop the trailing zeros to get the correct mantissa.**

4.509 x 10¹¹ **Step 4: Write the number in scientific notation as a product of the mantissa (from step 3) and power of 10 with correct exponent (from step 2).**

2. Write 0.0000563 in scientific notation.

0.00005.63 **Step 1: Move the decimal to a position immediately to the right of the first nonzero digit. Scan the number from left to right.**

0.00005.63 **Step 2: Count the number of place values the decimal point was moved. In this case, the decimal point was moved 5 spaces to the right. Therefore, the exponent on base 10 will be *negative* 5.**

~~00000~~5.63 **Step 3: Drop the leading zeros to get the correct mantissa.**

5.63 x 10⁻⁵ **Step 4: Write the number in scientific notation as a product of the mantissa (from step 3) and power of 10 with correct exponent (from step 2).**

Practice Questions:

57. Write 1,705,000 in scientific notation.

58. Write 0.0000807 in scientific notation.

Addition and Subtraction with Scientific Notation

To ADD or SUBTRACT numbers written in scientific notation:

- The numbers **MUST have the same exponent** on the powers of 10.
 - To **increase an exponent** in scientific notation, move the decimal point in the mantissa to the **left** the same number of times that you would like to increase the exponent. (For example, to increase the exponent by 2, add 2 to the exponent and move the decimal point in the mantissa to the left two times).
 - To **decrease an exponent** in scientific notation, move the decimal point in the mantissa to the **right** the same number of times that you would like to decrease the exponent. (For example, to decrease the exponent by 3, subtract 3 from the exponent and move the decimal point in the mantissa three times to the right).
- When all numbers in scientific notation have powers with the same exponent, **add or subtract the mantissa(s). Keep the power the same.**
- If necessary, adjust the mantissa and exponent** to put the final answer in proper scientific notation. Remember that the mantissa **MUST** be equal to or greater than 1 but less than 10.

Example:

Add 5.89×10^4 and 9.5×10^6 .

Step 1: The powers, 10^4 and 10^6 , have different exponents. We need to make these the same.

Method 1: *Re-write 9.5×10^6 as mantissa $\times 10^4$*

$$9.5 \times 10^6 = 950 \times 10^4 \quad \text{Decrease the exponent by 2; move the decimal point twice to the right in the mantissa.}$$

Step 2: Add the mantissas. Keep the powers the same.

$$\begin{aligned} 5.89 \times 10^4 + 950 \times 10^4 &= (5.89 + 950) \times 10^4 \\ &= 955.89 \times 10^4 \end{aligned}$$

Step 3: Adjust the mantissa and exponent to put the answer in proper scientific notation.

955.89 needs to be adjusted so that the mantissa is greater than or equal to 1 and less than 10.

Move the decimal point in the mantissa two times to the left and add 2 to the exponent.

$$955.89 \times 10^4 = 9.5589 \times 10^{4+2} = 9.5589 \times 10^6$$

←

Final answer is 9.5589×10^6 .

Method 2: Re-write 5.89×10^4 as mantissa $\times 10^6$

$$5.89 \times 10^4 = 0.0589 \times 10^6 \quad \text{Increase the exponent by 2; move the decimal point twice to the left in the mantissa.}$$

Step 2: Add the mantissas. Keep the powers the same.

$$\begin{aligned} 0.0589 \times 10^6 + 9.5 \times 10^6 &= (0.0589 + 9.5) \times 10^6 \\ &= 9.5589 \times 10^6 \end{aligned}$$

Step 3: The mantissa, 9.558, is greater than 1 and less than 10. The answer is in proper scientific notation.

Final answer is 9.5589×10^6 .

Example 2:

Subtract 9.53×10^8 and 1.2×10^7 .

Step 1: The powers, 10^8 and 10^7 , have different exponents. We need to make these the same.

Method 1: Re-write 9.53×10^8 as mantissa $\times 10^7$

$$9.53 \times 10^8 = 95.3 \times 10^7 \quad \text{Decrease the exponent by 1; move the decimal point once to the right in the mantissa.}$$

Step 2: Subtract the mantissas. Keep the powers the same.

$$\begin{aligned} 95.3 \times 10^7 - 1.2 \times 10^7 &= (95.3 - 1.2) \times 10^7 \\ &= 94.1 \times 10^7 \end{aligned}$$

Step 3: Adjust the mantissa and exponent to put the answer in proper scientific notation.

94.1 needs to be adjusted so that the mantissa is greater than or equal to 1 and less than 10.

Move the decimal point in the mantissa once to the left and add 1 to the exponent.

$$94.1 \times 10^7 = 9.41 \times 10^{7+1} = 9.41 \times 10^8$$

←

Final answer is 9.41×10^8 .

Method 2: Re-write 1.2×10^7 as mantissa $\times 10^8$

$1.2 \times 10^7 = 0.12 \times 10^8$ Increase the exponent by 1; move the decimal point once to the left in the mantissa.

Step 2: Subtract the mantissas. Keep the powers the same.

$$9.53 \times 10^8 - 0.12 \times 10^8 = (9.53 - 0.12) \times 10^8 \\ = 9.41 \times 10^8$$

Step 3: The mantissa, 9.41, is greater than 1 and less than 10. The answer is in proper scientific notation.

Final answer is 9.41×10^8 .

Practice Questions:

59. Add. $1.567 \times 10^3 + 3.2 \times 10^4$

60. Subtract. $7.85 \times 10^5 - 4.5 \times 10^3$

Multiplication and Division with Scientific Notation

To MULTIPLY numbers written in scientific notation:

1. Multiply the mantissas.
2. Add the exponents on powers of 10.
3. If necessary, adjust the mantissa and exponent to put the final answer in proper scientific notation.

Note: The numbers do NOT need to have the same exponent on the powers of 10.

Example:

Multiply 3.67×10^8 and 1.3×10^3 .

$$(3.67 \times 10^8)(1.3 \times 10^3) = (3.67 \times 1.3) \times 10^{8+3}$$

Step 1: Multiply the mantissas. Step 2: Add the exponents.

$$= 4.771 \times 10^{11}$$

Answer is in proper scientific notation.

To **DIVIDE** numbers written in scientific notation:

1. **Divide the mantissas.**
2. **Subtract the exponents** on powers of 10.
3. **If necessary, adjust the mantissa and exponent** to put the final answer in proper scientific notation.

Note: The numbers do NOT need to have the same exponent on the powers of 10.

Example:

Divide 9.5×10^8 and 5×10^4 .

$$(9.5 \times 10^8) \div (5 \times 10^4) = (9.5 \div 5) \times 10^{8-4}$$

Step 1: Divide the mantissas.

$$= 1.9 \times 10^4$$

Step 2: Subtract the exponents.

Answer is in proper scientific notation.

Practice Questions:

61. Multiply 1.87×10^3 by 5.193×10^4 .
62. Divide 7.707×10^5 by 2.1×10^2 .

10. Square Roots

The square root of a number is defined below:

$$\text{If } x^2 = y, \text{ then } \sqrt{y} = x$$

When finding the square root of a number, we are looking for a number that multiplied by itself would give us the number underneath the square root.

For example:

$$\sqrt{16} = ?$$

What number multiplied by itself would equal to 16? One such number is 4, since $4 \times 4 = 16$. Another possible number is -4 , since $(-4)(-4) = 16$. Thus, the square root of 16 is equal to 4 or -4 . Both of these are valid answers.

Practice question:

63. What is the $\sqrt{121}$?

- a) 11
- b) $\frac{121}{2}$
- c) 12
- d) -11
- e) Both a and d

From above, 16 and 121 are both examples of **perfect squares**.

Perfect squares are numbers whose square root is a whole number.

There is an infinite number of perfect squares. Some are listed below:

1, 4, 9, 16, 25, 36, 49, 64, 81, 100, 121, 144, 169, 196, 225, ...

It is good to remember and be able to recognize which numbers are perfect squares and which are not. Knowing the perfect squares can help to *estimate* the value of the square root of a number that is not a perfect square.

Example:

Estimate the value of $\sqrt{62}$.

62 lies between the two perfect squares 49 and 64. Thus, the square root of 62 would be somewhere between $7 (= \sqrt{49})$ and $8 (= \sqrt{64})$. Since 62 is much closer to 64 than to 49, the square root of 62 would be closer to 8 than to 7. A reasonable guess might be 7.8 or 7.9.

The actual value for $\sqrt{62} = 7.874007874...$

NOTE: Most calculators only give the positive square root of a number. Likewise, we usually only consider the positive root unless both the negative and positive root matters in our particular problem.

Practice Question:

64. Estimate the $\sqrt{78}$.
- a) Between 7 and 8.
 - b) Between 10 and 11.
 - c) Between 9 and 10.
 - d) Between 8 and 9.

It is possible to *evaluate* square roots of numbers that are not perfect squares by hand (not discussed here) or using a calculator.

On your calculator, use the “ $\sqrt{\quad}$ ” button to evaluate square roots.

Practice Question:

65. Evaluate the $\sqrt{28}$ using a calculator. Round to 1 decimal place.
- a) 6
 - b) 14
 - c) 5.3
 - d) -14

Solving One-variable Equations

Given an equation with one unknown variable, it is possible to *solve* the equation by determining the value of the variable that makes the equation true (i.e. left side of the equation equals the right side of the equation). The value of the variable can be determined from the given equation by isolating the variable to one side of the equation using opposite operations.

Opposite operations	
addition	subtraction
multiplication	division
exponent of 2 (squared)	square root ($\sqrt{\quad}$)
any positive exponent “n”	“nth” root ($\sqrt[n]{\quad}$)

Note: Equations with no solution and equations where the solution is all real numbers are not discussed here.

Example:

Solve for x.

$$2x + 4 = 10$$

$$2x + 4 = 10$$

Step 1: Eliminate + 4 from the left side (LS) by subtracting it. Whatever you do to the left side, you must do to the right side (RS). Therefore, subtract 4 from the RS, as well.

$$2x = 10 - 4$$

Step 2: Simplify by evaluating $10 - 4$ on the RS.

$$2x = 6$$

Step 3: Eliminate 2 from the LS by dividing the LS by 2. You must also divide the RS by 2.

$$x = \frac{6}{2}$$

Step 4: Evaluate the division.

$$x = 3$$

Final answer.

Example 2:

Solve for x.

$$\frac{2x - 18}{9} = -4$$

$$\frac{2x - 18}{9} = -4$$

Step 1: Eliminate 9 from the LS by multiplying both sides of the equation by 9.

$$2x - 18 = -4(9)$$

Step 2: Simplify the RS by evaluating the product of -4 and 9 .

$$2x - 18 = -36$$

Step 3: Eliminate -18 from the LS by adding 18 to both sides of the equation.

$$2x = -36 + 18$$

Step 3: Simplify the RS by evaluating the sum of -36 and 18 .

$$2x = -18$$

Step 4: Eliminate 2 from the LS of the equation by dividing both sides of the equation by 2.

$$x = \frac{-18}{2}$$

Step 5: Simplify the RS by evaluating the quotient of -18 and 2 .

$$x = -9$$

Final answer.

Practice Questions:

66. Solve for x.

$$-3x + 21 = 6$$

67. Solve for b.

$$\frac{7b + 23}{-2} = 27$$

11. Re-arranging Formulas

Tips for re-arranging formulas:

- The goal of re-arranging a formula is **to isolate the variable you want to one side of the equation**. All the other terms/variables in your formula should be on the other side of the equal sign.
- In order to move variables from one side of the equation to the other, **do the opposite operation** on the other side.

Example 1:

Isolate x in the formula $x + y = 1$

$$x + y = 1$$

Step 1: To isolate x , subtract y from both sides of the equation

$$x = 1 - y$$

Final answer.

Example 2:

Re-arrange the following formula to isolate y .

$$ax + by + c = 0$$

$$ax + by + c = 0$$

Step 1: Subtract ax and c from both sides of the equation to eliminate them from the left side. The term, by , should be all that remains on the left side of the equation.

$$by = -ax - c$$

$$by = -ax - c$$

Step 2: Divide both sides of the equation by b to eliminate it from the left side. y should be the only variable remaining on the left side

$$y = \frac{-ax - c}{b}$$

This is the final answer since y is isolated.

Sample question:

68. Re-arrange the following formula to isolate r .

$$rq^2 + 2.56s = t$$

- a) $r = t + 2.56s - q^2$
- b) $r = t - 2.56s + q^2$
- c) $r = q^2(t - 2.56s)$
- d) $r = \frac{t - 2.56s}{q^2}$

If the formula has fractions on both sides of the equal sign ($\frac{a}{b} = \frac{c}{d}$), use **cross-multiplication** to get rid of the fractions ($\frac{a}{b} = \frac{c}{d}$ becomes $ad = bc$).

Example:

Re-arrange the following formula to isolate x .

$$\frac{a+1}{x} = \frac{b}{2}$$

$$\frac{a+1}{x} \times \frac{2}{2} = \frac{b}{2} \times \frac{2}{2}$$

Step 1: Use cross-multiplication to get rid of fractions.

$$2(a+1) = bx$$

Step 2: Move b to the LS of the equation by dividing the LS by b .

$$\frac{2(a+1)}{b} = x$$

Final answer since x is isolated.

Sample question:

69. Re-arrange the following hypothetical formula to isolate “ z .”

$$\frac{3z+8y}{14} = a$$

- a) $z = 3(14a - 8y)$
- b) $z = \frac{8y - 14a}{3}$
- c) $z = \frac{14a - 8y}{3}$
- d) $z = 8ya + 14$

If the **variable that needs to be isolated is an exponent**, take **log** or **ln** on both sides to bring the variable down.

Definition of log:

If $10^x = y$, then $\log_{10}y = x$

Note: **log** is assumed to have a base 10.

Log Rule: **$\log(a^x) = x\log(a)$**

Definition of ln:

If $e^x = y$, then $\ln y = x$

Note: **ln** always has base e.

Example:

Isolate x in the formula is $y^x = p$

$$y^x = p$$

Step 1: Take **log** on both sides of the equation.

$$\log(y^x) = \log(p)$$



$$\log(y^x) = \log(p)$$

Step 2: The variable **x** in the **exponent** comes down in front of $\log(y)$ according to log rules.

$$x\log(y) = \log(p)$$

$$x\log(y) = \log(p)$$

Step 3: Divide both sides of the equation by $\log(y)$ to eliminate it from the left side. X should be the only variable remaining on the left side.

$$x = \frac{\log(p)}{\log(y)}$$

Final answer.

Example 2:

Re-arrange the following formula to isolate for n.

$$A = P(1 + i)^n$$

$$A = P(1 + i)^n$$

Step 1: Divide both sides of the equation by **P** to eliminate it from the right side.

$$\frac{A}{P} = (1+i)^n \quad \text{Step 2: Take } \ln \text{ on both sides of the equation.}$$

$$\ln\left(\frac{A}{P}\right) = n \ln(1+i) \quad \text{Step 3: Move the exponent, } n, \text{ so that it is written in front of the } \ln.$$

$$\ln\left(\frac{A}{P}\right) = n \ln(1+i) \quad \text{Step 4: Divide } \ln(1+i) \text{ from both sides of the equation to eliminate it from the right side. } n \text{ should be the only variable left on the right side.}$$

$$\frac{\ln\frac{A}{P}}{\ln(1+i)} = n \quad \text{This is the final answer since } n \text{ is isolated.}$$

Sample question:

70. Re-arrange the following hypothetical formula to isolate k.

$$d^k + c = ab$$

a) $k = \frac{ab - c}{d}$

b) $k = \frac{\log(d)}{\log(ab - c)}$

c) $k = d(ab + c)$

d) $k = \frac{\log(ab - c)}{\log(d)}$

Unit Conversions

Table 1: Conversion within the Imperial System		
Volume	Length/ Distance	Weight
1 pint = 16 fluid ounces	1 foot = 12 inches	1 pound (lb) = 16 ounces (oz)
1 quart = 2 pints	1 yard = 3 feet	1 ton = 2000 pounds
1 gallon = 4 quarts	1 mile = 1760 yards	
1 tablespoon (tbsp) = 3 teaspoons (tsp)	1 mile = 5280 feet	
1 cup (C) = 8 fluid ounces		
1 cup (C) = 16 tablespoons (tbsp)		

There are different ways to convert between different units of measurement. Two methods are presented below.

Method 1:

If moving from a smaller to a bigger unit, DIVIDE by the conversion factor.

Example:

When converting from ounces to pounds, divide the number of ounces given by 16 oz/lb.

Convert 20 ounces into pounds.

$$20 \text{ oz} \div 16 \text{ oz/lb} = 1.25 \text{ lb}$$

If moving from a bigger to a smaller unit, MULTIPLY by the conversion factor.

Example:

When converting from pounds to ounces, multiply the number of pounds given by 16 oz/lb.

Convert 3.2 pounds into ounces.

$$3.2 \text{ lb} \times 16 \text{ oz/lb} = 51.2 \text{ oz}$$

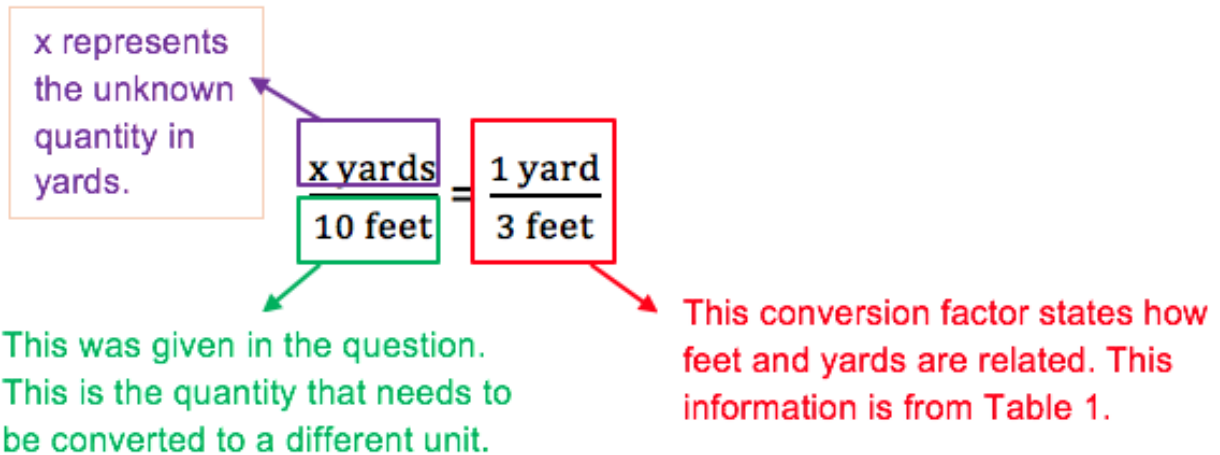
Method 2:

Proportion can be used to convert between different units of measurement.

Example:

Convert 10 feet into yards.

Step 1: Set up a proportion using the information given in the question and the conversion factor.



Step 2: Solve the proportion by isolating x.

2a. Cross multiply.

$$\frac{x}{10} \times \frac{1}{3}$$

$$3x = 10(1)$$

$$3x = 10$$

2b. To isolate x, move 3 to the right side of the equation; divide the right side by 3.

$$x = \frac{10}{3}$$

Step 3: Simplify by changing the improper fraction into a mixed number.

$$x = 3\frac{1}{3}$$

Therefore, 10 feet is equal to $3\frac{1}{3}$ yards.

Practice Questions:

71. Convert 12 teaspoons into tablespoons.

72. Convert 8 feet into yards.

73. Convert 3000 pounds into tons.

74. Convert 9 pounds into ounces.

75. Convert 6 pints into quarts.

Table 2: Conversions within the Metric system								
Units	kilo-	hecto-	deka-	base	deci-	centi-	milli-	micro-
Symbols	k	h	da	metre (m) gram (g) liter (L)	d	c	m	mc μ
Values compared to the base	1,000	100	10	1	0.1	0.01	0.001	0.000001

Conversions within the metric system are based on **powers of 10**. The **base units** are **metre, gram** and **litre**.

Units with the prefix **kilo-** are **1,000 times bigger** than the base unit. (E.g. 1 km = 1000 m)

Units with the prefix **hecto-** are **100 times bigger** than the base unit. (E.g. 1 hm = 100 m)

Units with the prefix **deka-** are **10 times bigger** than the base unit. (E.g. 1 dam = 10 m)

Units with the prefix **deci-** are **10 times smaller** than the base unit. (E.g. 10 dm = 1 m)

Units with the prefix **centi-** are **100 times smaller** than the base unit. (E.g. 100 cm = 1 m)

Units with the prefix **milli-** are **1,000 times smaller** than the base unit. (E.g. 1000 mm = 1 m)

Units with the prefix **micro-** are **1,000,000 times smaller** than the base unit.

Tips:

- To **multiply** a number by 10, move the decimal point once to the right.
- To multiply a number by 100, move the decimal point twice to the right.
- To multiply a number by 1000, move the decimal points three times to the right.
- To **divide** a number by 10, move the decimal point once to the left.
- To divide a number by 100, move the decimal point twice to the left.
- To divide a number by 1000, move the decimal point three times to the left.

Example:

Convert 50 L into mL.

Method 1: A litre is 1000 times bigger than a milliliter.

$$1 \text{ L} = 1000 \text{ mL}$$

To convert from a bigger to a smaller unit, multiply by the conversion factor.

To divide by 1000, move the decimal point three times to the right.

$$50 \text{ L} \times \frac{1000 \text{ mL}}{1 \text{ L}} = 50000 \text{ mL}$$

Method 2: Set up a proportion and solve for the unknown quantity.

$$\frac{x \text{ mL}}{50 \text{ L}} = \frac{1000 \text{ mL}}{1 \text{ L}}$$

$$x = 50 \times 1000$$

$$x = 50000 \text{ mL}$$

Therefore, 50 L is equal to 50000 mL.

Example 2:

Convert 345 dm into km.

Method 1: A kilometre is 10000 times bigger than a decimetre.

$$1 \text{ km} = 10000 \text{ dm}$$

To convert from a bigger to a smaller unit, divide by the conversion factor.

To divide by 10000, move the decimal point four times to the left.

$$345 \text{ dm} \div \frac{10000 \text{ dm}}{1 \text{ km}} = 0.0345 \text{ km}$$

Method 2: Set up a proportion and solve for the unknown quantity.

$$\frac{x \text{ km}}{345 \text{ dm}} = \frac{1 \text{ km}}{10000 \text{ dm}}$$

$$10000x = 345 \times 1$$

$$x = 345 \div 10000$$

$$x = 0.0345$$

Therefore, 345 dm is equal to 0.0345 km.

Practice Questions:

76. Convert 68 milliliters into liters.

77. Convert 345 dekametres into metres.

78. Convert 5.6 kilometres into metres.

79. Convert 450 grams into kilograms.

80. Convert 1.8 decimetres into millimetres.

Temperature Conversion Formulas

To convert Celsius to Fahrenheit: $(^{\circ}\text{C} \times \frac{9}{5}) + 32 = ^{\circ}\text{F}$

To convert Fahrenheit to Celsius: $(^{\circ}\text{F} - 32) \times \frac{5}{9} = ^{\circ}\text{C}$

Example:

Convert 32°C into $^{\circ}\text{F}$.

$(32^{\circ}\text{C} \times \frac{9}{5}) + 32 = ^{\circ}\text{F}$ Step 1: Perform the multiplication inside brackets.

$(\frac{288}{5}) + 32 = ^{\circ}\text{F}$ Step 2: Simplify $\frac{288}{5}$.

$57.6 + 32 = ^{\circ}\text{F}$ Step 3: Perform the addition.

$89.6 = ^{\circ}\text{F}$

Example 2:

Convert 98°F into $^{\circ}\text{C}$. (Round to one decimal place).

$(98 - 32) \times \frac{5}{9} = ^{\circ}\text{C}$ Step 1: Perform the subtraction inside brackets.

$(66) \times \frac{5}{9} = ^{\circ}\text{C}$ Step 2: Perform the multiplication.

$\frac{330}{9} = ^{\circ}\text{C}$ Step 3: Convert the fraction into a decimal.

$36.7 = ^{\circ}\text{C}$

Practice Questions

81. Convert 87°F into $^{\circ}\text{C}$. (Round the answer to 1 decimal place.)

82. Convert 22°C into $^{\circ}\text{F}$. (Round the answer to 1 decimal place.)

13. Measures of Central Tendency: Mean, Median, Mode

Mean

The mean for a given set of data is commonly referred to as the “**average**”. The mean is calculated by adding all of the values in a data set and dividing by the number of values there are in the set.

$$\text{Mean} = \frac{\text{sum of all data values}}{\text{number of values}}$$

Example:

Find the mean for the given data set.

7, -5, 16, 25, 9, 0, -3

There are 7 data values in the given data set.

$$\begin{aligned} \text{Mean} &= \frac{7+(-5)+16+25+9+0+(-3)}{7} \\ &= 7 \end{aligned}$$

Median

The median is the **middle value** in a set of data ordered from biggest to smallest or smallest to biggest values. Therefore, half of the values lie below the median and half lie above it.

If there is an *even* number of values in a data set, the median is the average of the middle two values.

Example:

Find the median for the given data set.

7, -5, 16, 25, 9, 0, -3, 15

Step 1: Order the data set from smallest to biggest values.

-5, -3, 0, 7, 9, 15, 16, 25

Step 2: Since there are 8 values in this data set, find the average of the two middle values.

-5, -3, 0, 7, 9, 15, 16, 25

$$\frac{7+9}{2} = \frac{16}{2} = 8$$

Therefore, the median for the data set is 8.

Mode

The mode is the **most frequently occurring value** in a given data set.

If there are NO values occurring more than once, a set of data has no mode.

There is more than one mode in a set of data if two or more values occur *equally frequently*.

Example:

Find the mode for the given data set.

9, 7, -5, 16, 25, 9, 0, -3, 15, 9, 13, 7

9, 7, -5, 16, 25, 9, 0, -3, 15, 9, 13, 7

9 occurs in this data set three times whereas all the other values occur two times or less. Therefore, 9 is the mode.

Practice Questions:

83. Find the mean, median and mode for the following data sets.

a) 19, 25, 36, 0, 25, 22, 18, 12

b) -9, 0, 15, 7, 2, -9, 15, 11

c) 134, 127, 98, 100, 155

Appendix A: Glossary

Conversion factor – a numerical factor used to multiply or divide a quantity when converting between different units

Decimal – a number that uses a decimal point followed by digits as a way of showing values less than one. The fractional part of a decimal is based on powers of 10

Decimal point – a dot or point used to separate the whole number from the fractional part in a decimal

Denominator – the “bottom” number in a fraction. The denominator indicates how many parts one whole is divided into

Difference – the result of subtraction

Dividend – the number that is being divided

Divisor – the number that a dividend is divided by

Equation – an equation uses an equal sign to state that two expressions are the same or equal to each other; (for example, $5x^2 + 18 = 25$)

Equivalent fractions – fractions that are equal in value. On a number line, equivalent fractions would occupy the same spot

Estimate – to give a reasonable guess

Evaluate – to calculate the numerical value

Exponent – the number in a power indicating how many times repeated multiplication is done

Factor – a number that will divide into another number exactly. (For example, factors of 6 are 1, 2, 3, and 6)

Greatest common factor – the largest number that is a factor of two or more given numbers. (For example, the greatest common factor of 27 and 36 is 9)

Improper fraction – a fraction where the numerator is greater than the denominator. An improper fraction is always greater than one whole

Integers – a set of numbers which is made up of positive whole numbers, zero and negative whole numbers.

Leading zeros – zeros that do not increase the value of a number but are used to fill place values.

Lowest common denominator – the lowest common denominator of two or more fractions is equal to the lowest common multiple of the fractions’ denominators

Lowest common multiple – the smallest number that is common in sets of multiples for two or more numbers (For example, the lowest common multiple of 3 and 4 is 12)

Lowest terms – a fraction is considered to be in lowest terms when it cannot be reduced any more. This occurs where there are no more common factors of the numerator and denominator other than 1

Mantissa – part of a number in **scientific notation** or a floating point, consisting of its significant digits

Mean – the mean is commonly referred to as the “**average**”. The mean is calculated by adding all of the values in a data set and dividing by the number of values there are

Median – the middle value in a set of data ordered from greatest to lowest or lowest to greatest

Mixed number – a number consisting of a whole number and a fraction

Mode – the most frequently occurring value in a given data set

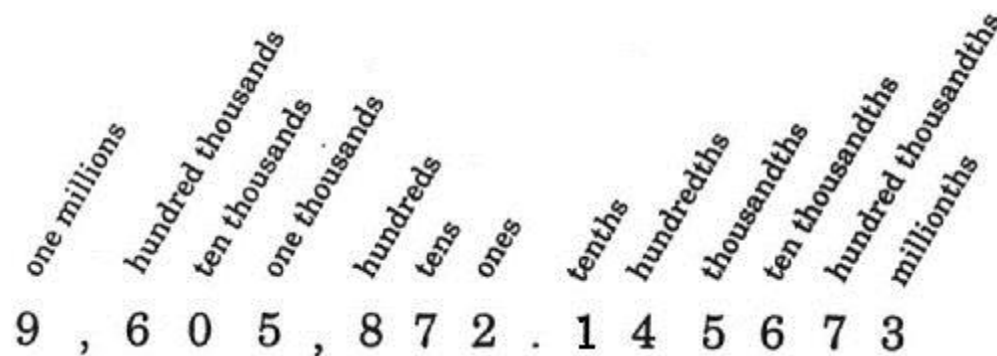
Multiple – the product of a number and any whole number. (For example, multiples of 3 are 3, 6, 9, 12, 15, 18, ...)

Numerator – the “top” number in a fraction. The numerator indicates how many parts of a whole the fraction represents

Percent – parts per 100

Perfect square – a number whose square root is a whole number

Place value – the location of a digit in a number and the specific name for that location



Source: <http://www.onlinemathlearning.com/place-value-chart.html>

Power – a number raised to an exponent

Product – the result of multiplication

Proper fraction – a fraction where the numerator is less than the denominator. A proper fraction is always less than one whole

Proportion – an equation stating that two ratios or two rates are equal to each other

Quotient – the result of division

Ratio – a comparison of two quantities with same units

Scientific notation – a method of writing numbers in terms of a decimal number between 1 and 10 multiplied by a power of 10. Scientific notation is usually used to write very small or very large numbers more compactly.

Significant digit – any digit of a number that is known with certainty

Simplify – to write an algebraic expression in simplest form; an expression is in simplest form when there are no more like terms that can be combined

Solve – to find the numerical value

Squared – raised to the exponent 2

Square root – the square root of a number is the value of the number, which multiplied by itself gives the original number

Sum – the result of addition

Trailing zeros – the zero(s) following the last nonzero digit of a number

Variable – a letter of the alphabet used to represent an unknown number or quantity

Appendix B: Multiplication Table

	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Appendix C: Rounding Numbers

Follow these steps for rounding numbers to a specified place value or decimal place.

1. Identify the directions for rounding. (E.g. Round to the nearest tenth. Round to two decimal places.)
2. Identify the number in the place value or the decimal place to be rounded to. Underline this number.
3. Look at the number to the **right** of the underlined number. (Note: Only look at ONE number immediately to the right.)
4. If the number to the right is 5 or higher, increase the underlined number by 1.
If the number to the right is 4 or lower, keep the underlined number the same. (Do NOT decrease the number).
5. If the underlined number is in the decimal place values, delete all numbers to the right of the underlined number.
If the underlined number is in the whole number place values, replace all numbers up to the decimal point with zeroes.

Example:

Round 5.8739 to two decimal places.

5.8739 (7 is the second decimal place)

5.8739 The number to the right of 7 is 3.
3 is less than 5; therefore, keep 7 as is.
Delete all numbers to the right of 7.

5.87 Final answer.

Example 2:

Round 14590 to the nearest thousand.

14590 (4 is in the thousands place)

14590 The number to the right of 4 is 5. Therefore, increase 4 by 1.
 $4 + 1 = 5$.
Replace all numbers to the right of 4 with zeroes.

15000 Final answer.

6. **Special case:** If the underlined number is a 9 and the number to the right is 5 or higher, the 9 would be rounded up to 10. In this case, the 1 is regrouped with the next number to the left of the underlined digit, and the underlined number is replaced with a 0.

Example:

Round 34.97 to one decimal place.

34.97 (9 is in the first decimal place)

34.97

The number to the right of 9 is 7. Therefore, increase 9 by 1.

$$9 + 1 = 10.$$

Regroup 1 with the next number to the left of 9, which is 4 in this case; replace 9 with a 0.

Delete all numbers to the right of one decimal place.

35.0

Final answer.

Appendix D: Answers to Practice Questions

Topic	Page Number	Question Number	Answer
Operations with Whole Numbers	3	1.	2422
	3	2.	26268
	4	3.	11468
	4	4.	50110
	6	5.	360088
	6	6.	744898
	7	7.	1524
	7	8.	9635
	9	9.	d
Operations with Integers	9	10.	4
	10	11.	-7
	10	12.	-23
	11	13.	25
	11	14.	-20
	11	15.	132
	11	16.	-27
	12	17.	c
	12	18.	a
Operations with Fractions	13	19.	d
	14	20.	b
	15	21.	c
	18	22.	b
	18	23.	c
	19	24.	b
	21	25.	c
	21	26.	a
	23	27.	d
	23	28.	a
	24	29.	d
Operations with Decimals	26	30.	37.239
	26	31.	1.1359
	27	32.	368.261
	27	33.	8.325
	28	34.	47.31919
	28	35.	0.39649

Operations with Decimals (...continued)	30	36.	18.9
	30	37.	1.653
Conversions between Fractions, Decimals and Percent	32	38.	b
	33	39.	d
	34	40.	c
Percent	35	41.	b
	35	42.	a
	35-36	43.	d
	37	44.	a
	38	45.	d
	39	46.	d
	39-40	47.	b
	41	48.	b
	41	49.	d
Ratio and proportion	41	50.	3:1
	42	51.	12
	42	52.	$1\frac{1}{2}$
	42	53.	2
	42	54.	$\frac{1}{2}$
Exponents	44	55.	d
	44	56.	d
Scientific Notation	43	57.	1.705×10^6
	43	58.	8.07×10^{-5}
	48	59.	3.3567×10^4
	48	60.	7.805×10^5
	49	61.	9.71091×10^7
	49	62.	3.67×10^3
Square Roots	50	63.	e
	50	64.	d
	51	65.	c
Solving One-Variable Equations	52	66.	$x = 5$
	52	67.	$b = -11$
Re-arranging Formulas	53	68.	d
	54	69.	c
	55	70.	d
Unit Conversions	57	71.	4 tbs
	57	72.	$2\frac{2}{3}$ yards
	57	73.	$1\frac{1}{2}$ tons

Unit Conversions (...continued)	57	74.	144 oz
	57	75.	3 quarts
	59	76.	0.068 L
	59	77.	3450 m
	59	78.	5600 m
	59	79.	0.45 kg
	59	80.	18 mm
	60	81.	30.6°C
	60	82.	71.6°F
Measures of Central Tendency	62	83.a)	Mean is 19.625 Median is 20.5 Mode is 25
	62	83.b)	Mean is 4 Median is 4.5 Mode is -9 and 15
	62	83.c)	Mean is 122.8 Median is 127 There is no mode